MAT267 Test 3 Study Topics and Practice (updated 11-16-25)

- Triple integrals in rectangular coordinates.

1. Evaluate these integrals:

$$\int_1^2 \int_{-1}^3 \int_0^4 x^2 y + z \, dz \, dy \, dx \qquad \int_0^4 \int_0^{2-\frac{1}{2}x} \int_0^{12-3x-6y} dz \, dy \, dx$$

2. Set up a triple integral in the $dz\ dy\ dx$ ordering that gives the volume below the plane whose axes intercepts are (6,0,0), (0,3,0) and (0,0,18), confined to the first octant.

- Cylindrical coordinates.

3. Convert to cylindrical coordinates and solve for an exact answer:

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{5-x-y} dz \, dy \, dx$$

4. Two paraboloids, $z = x^2 + y^2$ and $z = 16 - x^2 - y^2$, enclose a region in R^3. Set up a triple integral in cylindrical coordinates that gives the region's volume, and solve for the volume in exact form.

- Spherical coordinates.

5. Convert to spherical coordinates and solve exactly:

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \frac{1}{x^2 + y^2 + z^2} \, dz \, dy \, dx$$

6. A sphere centered at the origin, and a cone with its apex at the origin, meet such that the coordinate (4,3,2) lies on the lip where the two surfaces intersect. Set up an integral and solve for this object's volume exactly.

- Scalar line integrals.

7. Find $\int_C (2x + y) ds$ where C is the line segment from (1,2) to (5,4). State the answer in exact form.

8. Find $\int_C x \, ds$ where C is the parabola $y = x^2$ from (0,0) to (3,9).

9. Find $\int_C xy \, ds$ where C is the arc of the circle $x^2 + y^2 = 16$ traced counterclockwise from (4,0) to (0,4).

- Work line integrals. Some possibly may use the FTLI.

10. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle 2x, xy \rangle$ and C is the line segment from (2,3) to (7,1).

11. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle x - y, x^2 \rangle$ and C is the curve $y = x^2$ from (–1,1) to (5,25).

12. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle 1, xy \rangle$ and C is the quarter circle $x^2 + y^2 = 4$ from (2,0) to (0,2).

13. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle 3x^2 y, x^3 \rangle$ and C is the curve $y = \sqrt{2x + 1}$ for $0 \le x \le 4$.

14. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = \langle 2x+5,\ 3y^2-2\rangle$ and C is the path composed of line segments from (1,1) to (2,4) to (9,3) to (6,4).

15. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = \langle 3x+2y,\ 2x-4y\rangle$ and C is a sequence of line segments from (0,0) to(4,5) to (2,3) back to (0,0).

16. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = \langle 10x+11y,\ 8x-13y^2\rangle$ and C is the triangle with vertices (1,1) to (5,1) to (4,8) back to (1,1).

17. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = \langle 4x^2-3y,\ 6x+y^3\rangle$ and C is traced from (4,3) to (4,9) to (9,9) to (9,3) back to (4,3).

18. Vector field $\mathbf{F}(x,y) = \langle y^4, 4xy^3\rangle$ acts on a particle that moves from (1,4) to (5,7) to (3,10) back to (1,4).

    a. Find the work done for the segment from (1,4) to (5,7).
    b. Use part (a) to find the work done for the movement along the other two segments in the order stated above.

19. Find the work done on a particle that moves from (0,0) to (2,0) to (2,10) back to (0,0) within vector field $\mathbf{F}(x,y) = \langle x+y^2,\ 1+3x\rangle$.

Answers: (please post questions and possible errors to Ed Discussions) Errors to 9, 10 and 12 fixed.

1. $\frac{208}{3}$ and 16
2. $\int_0^6 \int_0^{3-\frac{1}{2}x} \int_0^{18-3x-6y} dz\, dy\, dx$
3. $\int_0^{2\pi} \int_0^2 \int_0^{5-r\cos\theta-r\sin\theta} dz\, r\, dr\, d\theta$, 20π
4. $\int_0^{2\pi} \int_0^{2\sqrt{2}} \int_{r^2}^{16-r^2} dz\, r\, dr\, d\theta$, 64π
5. This is a quarter sphere of radius 3 above the *xy*-plane and in front of the *yz* plane. The bounds are $0 \le \rho \le 3$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and $0 \le \phi \le \frac{\pi}{2}$. The integral is $\int_0^{\pi/2} \int_{-\pi/2}^{\pi/2} \int_0^3 \frac{1}{\rho^2} \rho^2 \sin\phi\ d\rho\, d\theta\, d\phi$. The ρ^2 cancel, so $\int_0^3 \sin\phi\ d\rho = 3\sin\phi$. The theta integral gives π, so we have $\int_0^{\pi/2} 3\pi \sin\phi\ d\phi = 3\pi[-\cos\phi]_0^{\frac{\pi}{2}} = 3\pi\big(0-(-1)\big) = 3\pi$.
6. The point's distance from the origin is $\sqrt{4^2+3^2+2^2} = \sqrt{16+9+4} = \sqrt{29}$ so $0 \le \rho \le \sqrt{29}$. The object goes all the way around the z-axis so $0 \le \theta \le 2\pi$, and the "lean" angle ranges from $0 \le \phi \le \arccos\frac{2}{\sqrt{29}}$. The integral is $\int_0^{\arccos(2/\sqrt{29})} \int_0^{2\pi} \int_0^{\sqrt{29}} \rho^2 \sin\phi\ d\rho\, d\theta\, d\phi = \frac{2\pi}{3}\big(\sqrt{29}\big)^3\big(1-2/\sqrt{29}\big)$.
7. $9\sqrt{20}$ or $18\sqrt{5}$.
8. $\frac{1}{12}(37^{1.5}-1)$
9. 32
10. 86/3
11. 282
12. 2/3
13. 192
14. 117
15. 0
16. –42
17. –270. Note: the original problem had point (9,4) as one vertex of the region, resulting in a trapezoid. The original problem could still be solved as 9 times the area of the trapezoid, which would be $\frac{1}{2}(5)(6+5) = 27.5$, so the value of the line integral would have been (9)(27.5) = 247.5, then negated to –247.5 since the region was traversed clockwise originally. In this case, the region is a rectangle, area (6)(5) = 30, so (9)(30) = 270, then the negation. Please be aware that this "trick" works only when the integrand is constant. Then you can multiply by the area of the region, using geometry. If the integrand is not constant, then you must actually work out a double integral, see #19.
18. (a) 11,749; (b) –11,749
19. –110/3