1. $\nabla f = \langle 2x, 3y^2 \rangle$

$\nabla f(1,2) = \langle 2, 12 \rangle$

unit vector of $\langle 4,3 \rangle$ is $\left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$

$\therefore\ D_w f = \langle 2, 12 \rangle \cdot \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle = \frac{8}{5} + \frac{36}{5} = \frac{44}{5}$

Other form: v = < 3/sqrt(10), 1/sqrt(10) >, so <2, 12> dot < 3/sqrt(10), 1/sqrt(10) > = (6+12)/sqrt(10) = 18/sqrt(10)

2. $\nabla f = \langle 15x^2 + 4y,\ 15y^2 + 4x \rangle$

$\nabla f(2,-1) = \langle 56, 23 \rangle$

$f(2,-1) = 27$

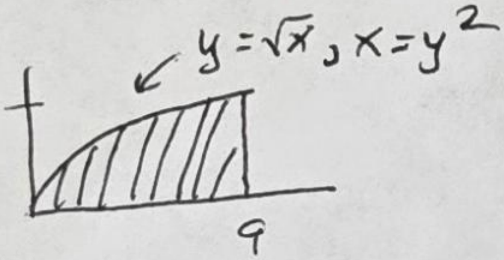
$0 = 56(x-2) + 23(y+1) - (z-27)$

$\rightarrow z = 56x + 23y - 62$

3. $$\int_0^2 \underbrace{\int_0^{2x} x^2 y \, dy}_{\frac{1}{2}y^2\Big]_0^{2x} = 2x^2} dx$$

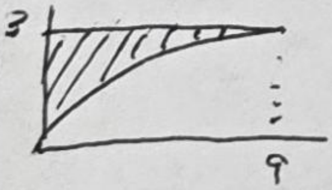
$$\int_0^2 x^2(2x^2)\,dx = \int_0^2 2x^4\,dx = \frac{2}{5}x^5\Big]_0^2 = \frac{64}{5}$$

4. $\int_0^9 \int_0^{\sqrt{x}} f(x,y)\,dy\,dx \Rightarrow$



$\Rightarrow \int_0^3 \int_{y^2}^9 f(x,y)\,dx\,dy$

$\int_0^9 \int_{\sqrt{x}}^3 f(x,y)\,dy\,dx \Rightarrow$



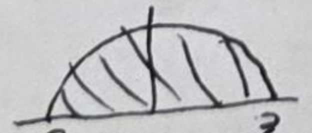
$\Rightarrow \int_0^3 \int_0^{y^2} f(x,y)\,dx\,dy$

5. Direction $= \nabla f = \langle y^2+2xy,\ 2xy+x^2 \rangle$

$\nabla f(3,2) = \langle 16, 21 \rangle$

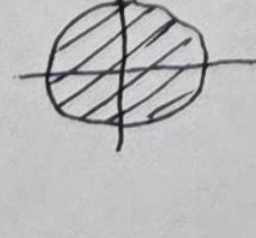
slope : $\sqrt{16^2+21^2} = \sqrt{697}$

6. $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} e^{x^2+y^2}\,dy\,dx$:



$\Rightarrow \int_0^{\pi} \int_0^3 r e^{r^2}\,dr\,d\theta$

$\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} e^{x^2+y^2}\,dy\,dx$



$\Rightarrow \int_0^{2\pi} \int_0^3 r e^{r^2}\,dr\,d\theta$

7.

$$\frac{\partial \cancel{z}}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

$10 = -4(?) + 5(-2)$

$20 = -4(?)$

$\Rightarrow \frac{\partial x}{\partial v} = -5$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$3 = -4(?) + 5(1)$

$-2 = -4(?)$

$\Rightarrow \frac{\partial x}{\partial u} = \frac{1}{2}$

8. $f(x,y) = x^{1/3} + y^{1/2}$ $\left\{\begin{matrix} x: \text{all } \mathbb{R} \\ y: y \geq 0 \end{matrix}\right\}$ QI, II

$f(x,y) = x^{1/2} + y^{1/2}$ $\left\{\begin{matrix} x: x \geq 0 \\ y: y \geq 0 \end{matrix}\right\}$ QI

9. $f(x,y) = x^3 + y^2 - 6xy$

$f_x = 3x^2 - 6y$

$f_y = 2y - 6x$

$\left.\begin{array}{l} 3x^2 - 6y = 0 \\ 2y - 6x = 0 \rightarrow y = 3x \end{array}\right.$

so $3x^2 - 18x = 0$

$3x(x-6) = 0$

$x = 0,\ x = 6$

$\rightarrow y = 0,\ y = 18$

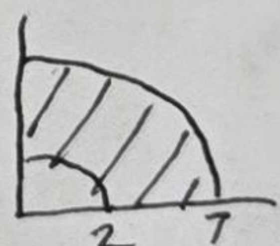
$\rightarrow z = 0,\ z = -108$

$D = f_{xx} f_{yy} - f_{xy}^2 = (6x)(2) - (-6)^2$

at $x = 0$: $D < 0$ $\therefore (0,0,0)$ saddle

$x = 6$: $D > 0$, $f_{xx} > 0$ $\therefore (6, 18, -108)$ MIN

10.



$\int_0^{\pi/2} \int_2^7 \underbrace{r^2 r}_{r^3} \, dr \, d\theta$

$\frac{1}{4} r^4 \Big]_2^7 = \frac{2385}{4}$

$\therefore \int_0^{\pi/2} d\theta = \frac{\pi}{2}$

$\frac{2385}{8}\pi$

$\int_0^{\pi/2} \int_2^7 r \cdot r \, dr \, d\theta$

$\frac{1}{3} r^3 \Big]_2^7 = \frac{335}{3} \rightarrow \frac{335}{6}\pi$

11. $A = 2\pi r h + \pi r^2$

$dA = A_r\, dr + A_h\, dh$

$dA = (2\pi h + 2\pi r)\, dr + (2\pi r)\, dh$

$= (2\pi(10) + 2\pi(8))(.05) + (2\pi(8))(.05)$

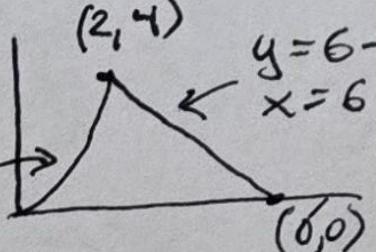
$= 36\pi(.05) + 16\pi(.05) = \frac{13}{5}\pi$

other film: r, h swapped: $\frac{14}{5}\pi$

12.

$y = x^2$

$x = \sqrt{y}$



$y = 6 - x$

$x = 6 - y$

$$\int_0^4 \int_{\sqrt{y}}^{6-y} g(x,y)\, dx\, dy$$