Quiz 9

4-10-25

① Evaluate $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F} = \langle 2xy, -3x \rangle$ and C is the curve $y = x^4$ from $(0,0)$ to $(3,81)$.

3 pts

Not conservative. $C: \vec{r} = \langle t, t^4 \rangle,\ 0 \le t \le 3.$ $d\vec{r} = \langle 1, 4t^3 \rangle.$ $\vec{F}(t) = \langle 2(t)(t^4), -3(t) \rangle = \langle 2t^5, -3t \rangle$

$\Rightarrow \vec{F}\cdot d\vec{r} = \langle 2t^5, -3t \rangle \cdot \langle 1, 4t^3 \rangle = 2t^5 - 12t^4 \Rightarrow \int_0^3 2t^5 - 12t^4\,dt = \frac{1}{3}t^6 - \frac{12}{5}t^5 \Big]_0^3 = -\frac{1701}{5}$ or -340.2

② Evaluate $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F} = \langle y^3 + 2, 3xy^2 - 2y \rangle$ and C is the curve $y = \sqrt{x}$ from $(1,1)$ to $(36,6)$, then a line to $(3,4)$.

2 pts

② Use FTLI: $f(x,y) = xy^3 + 2x - y^2 \rightarrow$ eval at end pts:

$(3,4) \rightarrow (3)(4)^3 + 2(3) - (4)^2 = 182$

$(1,1) \rightarrow (1)(1)^3 + 2(1) - (1)^2 = 2$

$182 - 2 = 180$

③ Evaluate $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F} = \langle \sqrt{x} + 2y, 5x + \frac{1}{y} \rangle$ and C is a path from $(1,1)$ to $(7,1)$ to $(5,9)$ back to $(1,1)$.

2 pts

Greens: $N_x - M_y = 5 - 2 = 3$

so $3\iint dA = 3(\text{area of triangle}) = 3\left(\frac{1}{2}(6)(8)\right) = 72$

④ Let $\vec{F} = \langle \frac{1}{y}, -\frac{x}{y^2} \rangle$

ⓐ Suppose path C is a triangle, where C_1 is from $(1,1)$ to $(3,1)$ to $(3,10)$. Evaluate $\int_{C_1} \vec{F}\cdot d\vec{r}$.

2 pts

ⓑ Suppose C_2 is from $(3,10)$ back to $(1,1)$. Use your answer in part ⓐ to evaluate $\int_{C_2} \vec{F}\cdot d\vec{r}$.

1 pt

a) use FTLI: $f(x,y) = \frac{x}{y}$ so $f(3,10) = \frac{3}{10}$, $f(1,1) = 1$, so $\frac{3}{10} - 1 = -\frac{7}{10}$

b) $\frac{7}{10}$