

# QUIZ 6 (3-4-25)

① [5 pts] Let  $f(x,y) = x^3 + y^2 - 27x - 8y$ . Find all critical points including the z-coordinate, and use the D-test to classify the points as a minimum, maximum or saddle.

② [5 pts] Find the <sup>local</sup> maximum and minimum values of the function in question 1 when constrained to the line  $y = x + 4$ . Hint: You will need Desmos or a calculator. State your answers as ordered triples, to 4-decimal places

Key: ①  $\nabla f = \langle 3x^2 - 27, 2y - 8 \rangle$   
 set to zero:  $3x^2 - 27 = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$   
 $2y - 8 = 0 \Rightarrow y = 4$   
 critical pts  $(3, 4, -70)$  and  $(-3, 4, 38)$   
 $D = (f_{xx})(f_{yy}) - (f_{xy})^2 \rightarrow (6x)(2) - 0^2$   
 $\rightarrow$  at  $x=3, y=4, D > 0, f_{xx} > 0 \therefore \text{MIN}$   
 $\rightarrow$  at  $x=-3, y=4, D < 0 \therefore \text{SADDLE}$

② sub  $y = x + 4$ :  
 $f(x, x+4) = x^3 + (x+4)^2 - 27x - 8(x+4)$   
 $\rightarrow = x^3 + x^2 - 27x - 16$   
 $f' = 3x^2 + 2x - 27$   
 set to 0:  $x = -3.3518$   
 $x = 2.68513$   
 crit pts:  
 $(-3.3518, 0.6482, 48.0772)$  max  
 $(2.68513, 6.68513, -61.929)$  min