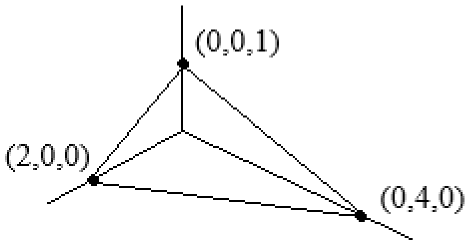
MAT267 Quiz December 3, 2024 **Name:**

Be neat. Submit to Canvas site by midnight on Wednesday December 4. One page PDF only. 5 pts each.

1. Let $\mathbf{F}(x, y, z) = \langle 2x, y, z\rangle$ be a vector field. Find the flux through the plane passing through (2,0,0), (0,4,0) and (0,0,1), where positive flow is in the direction of positive *z*.



Face: $\frac{x}{2} + \frac{y}{4} + z = 1 \to z = 1 - \frac{1}{2}x - \frac{1}{4}y$.

Normal vector $\mathbf{n} = \langle \frac{1}{2}, \frac{1}{4}, 1\rangle$.

$\mathbf{F}(x,y) = \langle 2x, y, 1 - \frac{1}{2}x - \frac{1}{4}y\rangle$.

$\mathbf{F} \cdot \mathbf{n} = \langle 2x, y, 1 - \frac{1}{2}x - \frac{1}{4}y\rangle \cdot \langle \frac{1}{2}, \frac{1}{4}, 1\rangle = x + \frac{1}{4}y + 1 - \frac{1}{2}x - \frac{1}{4}y = 1 + \frac{1}{2}x$.

Integrate as a *dy dx* function. The bounds are $0 \le y \le 4 - 2x$ and $0 \le x \le 2$.

$\int_0^2 \int_0^{4-2x} \left(1 + \frac{1}{2}x\right)\, dy\, dx = \int_0^2 \left[\left(1 + \frac{1}{2}x\right)(4 - 2x)\right] dx = \int_0^2 4 - x^2\, dx = \left[4x - \frac{1}{3}x^3\right]_0^2 = 8 - \frac{8}{3} = \frac{16}{3}$.

2. Use the divergence theorem to find the total net flow through the four-sided solid with planar faces and vertices (2,0,0), (0,4,0), (0,0,1) and (0,0,0). This is the same image as above, now including the parts on the *xy*, *yz* and *xz* planes. You may use geometry to solve. Do you remember the formula for the volume of a pyramid with a triangular base?

Div F $= \frac{\partial}{\partial x}(2x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z) = 2 + 1 + 1 = 4$.

$\iiint_S 4\, dV = 4 \iiint_S dV = 4(\text{volume of solid}) = 4\left(\frac{1}{3}\left(\frac{1}{2}bh\right)\right) = \frac{2}{3}(2)(4) = \frac{16}{3}$.

The volume of any pyramid is $V = \frac{1}{3}(\text{area of base})(\text{height})$.