MAT267 Quiz [redacted] **Name:**

Be neat. Submit to Canvas site by midnight on Wednesday [redacted]. One page PDF only. 5 pts each.

4-23-25

1. Find the surface area of the plane in the first octant passing through (2,0,0), (0,8,0) and (0,0,5). Exact answers only.

Plane is $\frac{x}{2}+\frac{y}{8}+\frac{z}{5}=1$, or $20x+5y+8z=40$.

Solved for z, we get $z=-\frac{5}{2}x-\frac{5}{8}y+5$.

The normal will be $\mathbf{n}=\langle\frac{5}{2},\frac{5}{8},1\rangle$.

Its magnitude is $\sqrt{\left(\frac{5}{2}\right)^2+\left(\frac{5}{8}\right)^2+1}=\frac{\sqrt{489}}{8}$.

The area is $\frac{\sqrt{489}}{8}\iint_R dA$ where R is the triangular region in the xy-plane, and its area is $\frac{1}{2}(2)(8)=8$, so the surface area of the planar portion is $8\left(\frac{\sqrt{489}}{8}\right)=\sqrt{489}$.

2. Find the surface area of the paraboloid $z=16-x^2-y^2$ confined to the first octant. Exact answers only.

The paraboloid is $\mathbf{r}(x,y)=\langle x,y,16-x^2-y^2\rangle$, its partial derivatives are $\mathbf{r}_x=\langle 1,0,-2x\rangle$ and $\mathbf{r}_y=\langle 0,1,-2y\rangle$. Cross product is $\mathbf{r}_x\times\mathbf{r}_y=\langle 2x,2y,1\rangle$,

The magnitude of the cross product is $\left|\mathbf{r}_x\times\mathbf{r}_y\right|=\sqrt{(2x)^2+(2y)^2+1^2}=\sqrt{4x^2+4y^2+1}$.

The paraboloid intersects the *xy*-plane ($z=0$) at a circle of radius 4, centered at the origin, so that the region of integration R is given by $x^2+y^2\le 16$. Remember, this is all in the first octant. Use polar coordinates:

$$\iint_R \sqrt{4x^2+4y^2+1}\,dA=\int_0^{\pi/2}\int_0^4\sqrt{4r^2+1}\,r\,dr\,d\theta.$$

The inside integral is:

$$\int_0^4\sqrt{4r^2+1}\,r\,dr=\left[\frac{1}{12}(4r^2+1)^{3/2}\right]_0^4=\frac{1}{12}\left(65^{3/2}-1\right).$$

Then, the outside integral is evaluated:

$$\frac{1}{12}\left(65^{3/2}-1\right)\int_0^{\pi/2}d\theta=\frac{\pi}{24}\left(65^{3/2}-1\right).$$