Test Yourself

For Questions 1-20, let vectors $\mathbf{u} = \langle 2,1,-3\rangle, \mathbf{v} = \langle 5,4,2\rangle$ and $\mathbf{w} = \langle -4,1,6\rangle$ be given. Find each of the following. If the answer does not exist, explain why.

1. $\mathbf{u} \cdot \mathbf{v}$.
2. $\mathbf{v} \cdot \mathbf{w}$.
3. $(\mathbf{u} \cdot \mathbf{w})\mathbf{v}$.
4. $\mathbf{v} \times \mathbf{w}$.
5. $\mathbf{u} + \mathbf{v} \cdot \mathbf{w}$.
6. $\mathbf{u} \cdot \mathbf{v} \cdot \mathbf{w}$.
7. $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$.
8. $(\mathbf{v} \cdot \mathbf{u}) \times \mathbf{w}$.
9. $2\mathbf{u} + 3\mathbf{v} - \mathbf{w}$.
10. $|\mathbf{u}|$.
11. The angle in degrees between $\mathbf{u}$ and $\mathbf{w}$.
12. A vector parallel to $\mathbf{v}$, but of length 2.
13. A vector parallel to $\mathbf{w}$, pointed in the opposite direction, of length 3.
14. The projection of $\mathbf{u}$ onto $\mathbf{v}$.
15. The projection of $\mathbf{v}$ onto $\mathbf{w}$.
16. The equation of the plane that contains $\mathbf{u}$ and $\mathbf{v}$ and includes the point (2,0,1).
17. The acute angle that vector $\mathbf{w}$ makes with the plane from Exercise 16.
18. The area of the parallelogram formed by $\mathbf{u}$ and $\mathbf{v}$.
19. The area of the triangle formed by $\mathbf{v}$ and $\mathbf{w}$.
20. Are $\mathbf{u}$ and $\mathbf{v}$ acute or obtuse?
21. A sphere has points (2, 5, 3) and (7, 9, 10) directly opposite one another. Find the equation of this sphere.

22. A sphere has center (4,7,2). Find the equation of the sphere with the largest possible radius such that it is fully contained within the first octant.

23. Find the radius of the circle that the sphere $(x-1)^2+(y+2)^2+(z-4)^2=25$ makes when it intersects the *xy* plane.

24. Find the equation of the line in R^3 that passes through (4, 2, –6) and is parallel to $\mathbf{u}=\langle 2,-1,5\rangle$.

25. The points A = (1,0,2), B = (4,1,1), C = (6,3,1) and D = (10,5,4). Find the distance from point D to the plane formed by A, B and C.

26. If vector **u** points north and **v** points southeast, then $\mathbf{u}\times\mathbf{v}$ points which way?

27. Find the equation of the plane equidistant between the points (1,3,2) and (4,–8,1).

28. Find the acute angle that the planes $x-2y+3z=6$ and $2x+y-7z=0$ meet.

29. Let $\mathbf{r}(t)=\langle t^2, 2t-3, 4t-t^3\rangle$ be a curve in space traced out by an object. Find $\mathbf{v}(3)$ and $\mathbf{a}(3)$, then find the object's speed at t = 5.

30. Find the equation of the tangent line to the curve in the previous in parametric form when t = 1.

31. Find the arc length of $\mathbf{r}(t)=\langle t, 3\sin t\,, 3\cos t\rangle$ for $0\le t\le 2\pi$.

32. Find the arc length of $\mathbf{r}(t)=\langle t^3, t\rangle$ for $-1\le t\le 2$.

33. Find the curvature of $y=x^3$ at $x=1$.

34. Let $\mathbf{a}(t)=\langle 0,2,t\rangle, \mathbf{v}(0)=\langle 1,2,1\rangle$ and $\mathbf{r}(0)=\langle -3,0,2\rangle$. Find $\mathbf{r}(1)$.

35. You start at (5,1) and start walking toward (1,6), intending to visit your significant other's home at (3,7). If you are allowed one right-angle turn, find the point at which you should make this turn so as to arrive at your sigoth's place in time for cartoons.

36. A force of 100 N hangs in the center of a cable between two anchor points that are the same height off the ground and 30 m apart horizontally. The object creates a sag of 2 m in the cable. Find the force exerted by each end of the cable on its anchor point.

37. A rock is propelled off a pedestal that is 10 meters off the level ground. The rock leaves the pedestal with a speed of 18 meters per second at an angle above the horizontal of 20 degrees. How high does the rock get, and how far downrange from the pedestal does the rock land?

38. An object rotates counterclockwise around a center at a distance of 5 m, making a revolution every 20 seconds. Assume it starts at (5,0). Find parametric equations that describe this object's position.

39. Classify the following surfaces as spheres, ellipsoids, hyperboloids of one or two sheets, cylinders, etc.

    a. $x^2 + y^2 - z^2 + 3x + y + 2z = 10$
    b. $x^2 + y^2 + z^2 + 5x + y + 4z = 8$
    c. $x^2 + 2y^2 + z^2 + x + 8y + 2z = 20$
    d. $x^2 + y^2 + 4x + y = 99$
    e. $x^2 - y^2 - 4z^2 + 3x + y + 12z = 31$
    f. $2x^2 + 2y^2 + 2z^2 + 3x + y + 2z = 80$

Answers.

1. 8
2. –4
3. $\langle -125, -100, -50 \rangle$
4. $\langle 22, -38, 21 \rangle$
5. Impossible, $\mathbf{v} \cdot \mathbf{w}$ is a scalar, $\mathbf{u}$ a vector, vector-to-scalar addition is not defined.
6. Impossible, $\mathbf{u} \cdot \mathbf{v}$ is a scalar, and the dot product of a scalar to a vector is not defined.
7. $\langle -117, -96, -62 \rangle$
8. Impossible, $\mathbf{v} \cdot \mathbf{u}$ is a scalar, and the cross product of a scalar to a vector is not defined.
9. $\langle 23, 13, -6 \rangle$
10. $\sqrt{14}$
11. 156.6 degrees
12. $\langle \frac{10}{\sqrt{45}}, \frac{8}{\sqrt{45}}, \frac{4}{\sqrt{45}} \rangle$
13. $\langle \frac{12}{\sqrt{53}}, -\frac{3}{\sqrt{53}}, -\frac{18}{\sqrt{53}} \rangle$
14. $\langle \frac{8}{9}, \frac{32}{45}, \frac{16}{45} \rangle$
15. $\langle \frac{16}{53}, -\frac{4}{53}, -\frac{24}{53} \rangle$
16. $14x - 19y + 3z = 31$
17. 19.214 degrees
18. $\sqrt{566}$
19. $\frac{1}{2}\sqrt{2369}$
20. Acute, since their dot product is positive.
21. $\left(x - \frac{9}{2}\right)^2 + (y - 7)^2 + \left(z - \frac{13}{2}\right)^2 = \frac{45}{2}$
22. $(x - 4)^2 + (y - 7)^2 + (z - 2)^2 = 4$
23. $r = 3$
24. $x(t) = 4 + 2t,\ y(t) = 2 - t,\ z(t) = -6 + 5t$
25. $\frac{4}{3}\sqrt{6} \approx 3.266$
26. Into the page.
27. $6x - 22y - 2z = 67$
28. 40.2 degrees.
29. $\mathbf{v}(3) = \langle 6, 2, -23 \rangle$; $\mathbf{a}(3) = \langle 2, 0, -18 \rangle$, $|\mathbf{v}(5)| = |\langle 10, 2, -71 \rangle| = \sqrt{5145} \approx 71.73$ units/time.
30. When t = 1, the position is $\mathbf{r}(1) = \langle 1, -1, 3 \rangle$, which can be treated as a point (1, –1, 3), and the velocity (tangent) vector is $\mathbf{v}(1) = \langle 2, 2, 1 \rangle$. Thus, the tangent line is $\langle 1 + 2t, -1 + 2t, 3 + t \rangle$.

31. $\sqrt{10} \cdot 2\pi$ units.
32. About 10.178 units.
33. $6/10^{3/2} \approx 0.19$.
34. $r(1) = \langle -2, 3, \frac{19}{6} \rangle$
35. $x = 5 - \frac{152}{41} \approx 1.293, y = 1 + \frac{190}{41} \approx 5.634$
36. About 378 N.
37. The rock reaches its highest point at t = 0.628 seconds, with a height of 11.93 meters, and the rock lands t = 2.189 seconds after being released, with a horizontal distance of 37.02 meters.
38. $\mathbf{r}(t) = \langle 5\cos\frac{2\pi}{20}t\,, 5\sin\frac{2\pi}{20}t \rangle$, or $\mathbf{r}(t) = \langle 5\cos\frac{\pi}{10}t\,, 5\sin\frac{\pi}{10}t \rangle$ when simplified.
39. (a) Hyperboloid of 1 sheet; (b) sphere; (c) Ellipsoid; (d) circular cylinder; (e) hyperboloid of 2 sheets; (f) sphere