MAT267 Test 1 – February 12, 2026 – Surgent **KEY**

Part I: Short answer. 6 points each, on a 6-3-0 scale: 6 means correct, 3 means one small error, 0 means incorrect.

1. Find the acute angle (in degrees) between the vectors $\mathbf{u} = \langle 4,2,1\rangle$ and $\mathbf{v} = \langle -1,5,7\rangle$.

A: $\theta = \cos^{1}\left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right) = \cos^{-1}\left(\frac{13}{\sqrt{21}\sqrt{75}}\right) = 70.88°$

Form B: $\theta = \cos^{-1}\left(\frac{1}{\sqrt{21}\sqrt{75}}\right) = 88.56°$

2. Find the equation of the line <u>segment</u> from $A = (4,1,5)$ to $B = (-4,2,7)$. Include bounds on t.

Form A: Direction vector: $AB = \langle -8,1,2\rangle$.
Line: $\langle 4,1,5\rangle + t\langle -8,1,2\rangle$ or $\langle 4-8t, 1+t, 5+2t\rangle$
$0 \le t \le 1$

Form B: Direction vector: $AB = \langle -8,1,-2\rangle$
Line: $\langle 4,1,7\rangle + t\langle -8,1,-2\rangle$ or $\langle 4-8t, 1+t, 7-2t\rangle$
$0 \le t \le 1$

3. Let $\mathbf{u} = \langle 2,0,1\rangle$ and $\mathbf{v} = \langle 4,1,3\rangle$. Give the area of the parallelogram formed by these two vectors.

$\mathbf{u}\times\mathbf{v} = \langle -1,-2,2\rangle$,
so $|\mathbf{u}\times\mathbf{v}| = \sqrt{(-1)^2+(-2)^2+2^2} = \sqrt{9} = 3$

4. Give the interval of k such that the vectors $\mathbf{u} = \langle 4,k,-1\rangle$ and $\mathbf{v} = \langle k,3,3\rangle$ are obtuse/acute.

Obtuse: $\mathbf{u}\cdot\mathbf{v} < 0$: $4k + 3k - 3 < 0 \to 7k < 3 \to k < \frac{3}{7}$

Acute: $\mathbf{u}\cdot\mathbf{v} > 0$: $4k + 3k - 3 > 0 \to 7k > 3 \to k > \frac{3}{7}$

5. If **u** and **v** are parallel non-zero vectors, then their dot product $\mathbf{u}\cdot\mathbf{v}$ is (circle the right answer)

   a. The zero vector
   b. The number 0
   c. $|\mathbf{u}||\mathbf{v}|$
   d. Not defined

Form A: c is the correct answer. Form B: a is correct.

6. A horizontal force of 6 N is applied to an object moving up a ramp of length 3 m at an angle of 15°. Find the work in Joules.

Form A: $\mathbf{F} = \langle 6,0\rangle, \mathbf{d} = \langle 3\cos 15, 3\sin 15\rangle$, so that
$\mathbf{F}\cdot\mathbf{d} = 6(3\cos 15) = 18\cos 15 \approx 17.39$ J.

Form B: $\mathbf{F} = \langle 5,0\rangle, \mathbf{d} = \langle 4\cos 25, 4\sin 25\rangle$, so that
$\mathbf{F}\cdot\mathbf{d} = 5(4\cos 25) = 20\cos 25 \approx 18.13$ J.

7. Where does the line $\langle 1+2t, 3t, 2-t\rangle$ intersect the plane $4x - 2y + z = 9$?

Form A: $4(1+2t) - 2(3t) + (2-t) = 9$, simplifies to $t + 6 = 9 \to t = 3$, so the point is $(7,9,-1)$

Form B: $4(1+2t) - 2(3t) + (2-t) = 8$, simplifies to $t + 6 = 8 \to t = 2$, so the point is $(5,6,0)$

8. An object's displacement is $\mathbf{r}(t) = \langle t^3, 2t^2 + 4t, \frac{1}{t}\rangle$. Find the object's speed at $t = 1$ second. Leave answer in exact (radical) form.

Form A: $\mathbf{r}'(t) = \langle 3t^2, 4t+4, -\frac{1}{t^2}\rangle$, so $\mathbf{r}'(1) = \langle 3,8,-1\rangle$.
Thus, speed = $|\mathbf{r}'(1)| = \sqrt{3^2+8^2+(-1)^2} = \sqrt{74}$.

Form B: $\mathbf{r}'(t) = \langle 4t^3, 6t+4, -\frac{1}{t^2}\rangle$,
so $\mathbf{r}'(1) = \langle 4,10,-1\rangle$.
Thus, speed = $|\mathbf{r}'(1)| = \sqrt{4^2+10^2+(-1)^2} = \sqrt{117}$.

Part II: Free response. Show all work and be NEAT!

9. Find the equation of the plane passing through the points $A = (3,2,1)$, $B = (-5,1,3)$ and $C = (0,-4,4)$. Leave answer in $ax + by + cz = d$ form.

$$\mathbf{AB} = \langle -8,-1,2\rangle, \mathbf{AC} = \langle -3,-6,3\rangle, \qquad \mathbf{n} = \mathbf{AB} \times \mathbf{AC} = \langle 9,18,45\rangle$$

Thus, the plane is $9(x - x_0) + 18(y - y_0) + 45(z - z_0) = 0$. Sub in any of the points:

$$9(x-3) + 18(y-2) + 45(z-1) = 0$$

Distribute and collect terms, move constant to the right:

$$9x - 27 + 18y - 46 + 45z - 45 = 0$$
$$9x + 18y + 45z = 108$$

Simplifies to

$$x + 2y + 5z = 12.$$

10. An object starts at point $A = (4,2)$ and moves in the direction of $\mathbf{v} = \langle 1,1\rangle$. The object is to finish at point $B = (10,4)$. If the object is allowed one right-angle turn, find the location of this turn.

Form A: The hypotenuse vector is $\mathbf{u} = \mathbf{AB} = \langle 10-4, 4-2\rangle = \langle 6,2\rangle$. Project this onto $\mathbf{v}$:

$$\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\mathbf{v} = \frac{8}{2}\langle 1,1\rangle = \langle 4,4\rangle.$$

Add to the original location: $\langle 4,2\rangle + \langle 4,4\rangle = \langle 8,6\rangle$ or $(8,6)$.

Form B: The hypotenuse vector is $\mathbf{u} = \mathbf{AB} = \langle 9-2,\ 7-4\rangle = \langle 7,3\rangle$. Project this onto $\mathbf{v}$:

$$\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\mathbf{v} = \frac{17}{5}\langle 2,1\rangle = \langle \frac{34}{5}, \frac{17}{5}\rangle = \langle 6.8, 3.4\rangle.$$

Add to the original location: $\langle 2,4\rangle + \langle 6.8,3.4\rangle = \langle 8.8, 7.4\rangle$ or $(8.8,7.4)$.

11. An object's position is given by $\mathbf{r}(t) = \langle 1 + \cos(2t), \sin(2t), 3t\rangle$.

a. Find the velocity vector at $t = \pi$.

$\mathbf{v}(t) = \mathbf{r}'(t) = \langle -2\sin(2t), 2\cos(2t), 3\rangle$, so $\mathbf{v}(\pi) = \langle -2\sin(2\pi), 2\cos(2\pi), 3\rangle = \langle 0,2,3\rangle$.

b. Find the unit tangent vector at $t = \pi$.

$$\text{Form A: } \mathbf{T}(\pi) = \frac{\mathbf{r}'(\pi)}{|\mathbf{r}'(\pi)|} = \langle 0, \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\rangle.$$

c. How far has the object travelled (the arc length) over the interval $0 \le t \le \pi$? Leave answer in exact form.

$$\text{Form A: } s = \int_0^{\pi} \sqrt{(-2\sin(2t))^2 + (2\cos(2t))^2 + 3^2}\, dt = \int_0^{\pi} \sqrt{13}\, dt = \pi\sqrt{13}.$$

Form B: the only change was the bound $\frac{\pi}{2}$. The answers are $\mathbf{v}\left(\frac{\pi}{2}\right) = \langle -2\sin(\pi), 2\cos(\pi), 3\rangle = \langle 0,-2,3\rangle$, $\mathbf{T}\left(\frac{\pi}{2}\right) = \langle 0, -\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\rangle$ and $s = \frac{\pi}{2}\sqrt{13}$.

12. (Form A) Points $A = (1,4,2)$ and $B = (-3,10,12)$ lie on a sphere exactly opposite one another (the line connecting them forms a diameter). Give the equation of this sphere in $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$ format.

Midpoint is (-1,7,7). Distance from midpoint to A or B is $\sqrt{37}$, so the sphere is $(x + 1)^2 + (y - 7)^2 + (z - 7)^2 = 37$.

(Form B): Find the point where lines $L_1 = \langle 1 + t, 2 + 4t, 4 - 3t \rangle$ and $L_2 = \langle 5 + s, 10 - 4s, -2 + 3s \rangle$ intersect.

Set the components equal and simplify:

$$\begin{matrix} 1 + t = 5 + s \\ 2 + 4t = 10 - 4s \\ 4 - 3t = -2 + 3s \end{matrix} \quad \rightarrow \quad \begin{matrix} -s + t = 4 \\ 4s + 4t = 8 \\ -3s - 3t = -6 \end{matrix} \quad \rightarrow \quad \begin{matrix} -s + t = 4 \\ s + t = 2 \\ s + t = 2 \end{matrix}$$

Solve the first two as a system:

$$\begin{matrix} -s + t = 4 \\ s + t = 2 \end{matrix} \rightarrow \text{(add columns)} \rightarrow 2t = 6 \rightarrow t = 3. \quad \text{This means } s = -1.$$

Check that it solves the third equation (it does).

Thus, they cross at $L_1 = \langle 1 + (3), 2 + 4(3), 4 - 3(3) \rangle = \langle 4, 14, -5 \rangle$.

Note that $s = -1$ plugged into L_2 gives the same vector/point.