

Quiz 5, due Friday Feb 20 at 11:59pm.

Consider the function $z = f(x,y) = x^2y + 3xy^2$.

1. Find the equation of the tangent plane at $x_0 = -2, y_0 = 3$. Leave your answer in $z = Ax + By + C$ form (z is isolated). (4pts)

2. Use the tangent plane approximation to give an estimation of $f(-1.9, 3.1)$. (3 pts)

3. A storage shed is in the shape of a pyramid with a square base. The volume of the pyramid is $V(b,h) = (1/3)b^2h$, where b is the length of one side of the square base, and h is the height of the pyramid. You measure the length of each side of the base to be 15 feet, and the height to be 12 feet, both with an accuracy of 0.04 feet. Use differentials to find the possible change in the volume when taking into account the accuracy threshold. (3 pts)

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$$f(x, y) = x^2 y + 3xy^2, \quad x_0 = -2, \quad y_0 = 3$$

$$z_0 = f(-2, 3) = (-2)^2(3) + 3(-2)(3)^2 = -42$$

$$\begin{aligned} \textcircled{1} \quad \vec{n} &= \langle f_x(x_0, y_0), f_y(x_0, y_0), -1 \rangle \\ &= \langle 2xy + 3y^2, x^2 + 6xy, -1 \rangle \\ &= \langle 2(-2)(3) + 3(3)^2, (-2)^2 + 6(-2)(3), -1 \rangle \\ &= \langle 15, -32, -1 \rangle \end{aligned}$$

$$\begin{aligned} \therefore \text{plane: } 15(x+2) - 32(y-3) - (z+42) &= 0 \\ 15x + 30 - 32y + 96 - z - 42 &= 0 \\ 15x - 32y - z + 84 &= 0 \end{aligned}$$

$$\therefore z = 15x - 32y + 84$$

$$\textcircled{2} \quad f(-1.9, 3.1) \approx 15(-1.9) - 32(3.1) + 84 = 43.7$$

(the actual point is $(-1.9, 3.1, -43.586)$)

$$\begin{aligned} \textcircled{3} \quad V &= \frac{1}{3} b^2 h \\ dV &= \frac{2}{3} b h db + \frac{1}{3} b^2 dh \\ &= \frac{2}{3} (15)(12)(0.04) + \frac{1}{3} (15)^2 (0.04) \\ &= 120(0.04) + 75(0.04) \\ &= 195(0.04) \end{aligned}$$

$$= 7.8 \text{ cubic feet}$$