MAT267 Quiz 3 – Due tonight (2/3/26) at 10pm, no exceptions. PDF only. 10 pts

1. Where does the line $\langle 5-t, 1+2t, 6t\rangle$ intersect the plane $3x-y+5z=64$? (4pts)

2. An object moves according to the function $\mathbf{r}(t)=\langle \cos(3t), 1+\sin(3t), 4t\rangle$, in meters. (2pts each part)

   a. Find the object's velocity vector at $t=2\pi$ seconds. Leave answer in reduced exact form.

   b. Find the unit tangent vector at this same *t* value.

   c. Find the distance (arc length) that the object travelled from $t=0$ to $t=2\pi$ seconds. Leave answer in exact form, no decimals.

Key:

1. Substitute: $3(5-t)-(1+2t)+5(6t)=64$. Simplify: $15-3t-1-2t+30t=64$. Collect terms and solve for t: $25t+14=64 \rightarrow 25t=50 \rightarrow t=2$. Thus, the point is $\langle 5-(2), 1+2(2), 6(2)\rangle = \langle 3,5,12\rangle$ or (3,5,12), written as a point or vector is fine.

2. Differentiate: $\mathbf{v}(t)=\mathbf{r}'(t)=\langle -3\sin(3t), 3\cos(3t), 4\rangle$.

   a. Velocity: $\mathbf{v}(2\pi)=\langle -3\sin(6\pi), 3\cos(6\pi), 4\rangle = \langle 0,3,4\rangle$. This is the only correct form to be accepted.

   b. $\mathbf{T}(2\pi)=\frac{\mathbf{r}'(2\pi)}{|\mathbf{r}'(2\pi)|}=\frac{\langle 0,3,4\rangle}{5}=\langle 0,\frac{3}{5},\frac{4}{5}\rangle$.

   c. $s=\int_0^{2\pi}|\mathbf{r}'(2\pi)|\,dt=\int_0^{2\pi}5\,dt=5(2\pi)=10\pi$. This is the only form to be accepted.