Line Integrals Practice

No answers are provided. If you want to discuss them, go to Piazza and post your work there for feedback. I want to encourage such discussions on Piazza.

You are given a vector field $\mathbf{F}(x, y) = \langle M(x, y), N(x, y)\rangle$ and a path in the *xy*-plane $C: r(t) = \langle x(t), y(t)\rangle$, where $a \leq t \leq b$. The following integrals,

$$\int \mathbf{F} \cdot \mathbf{T}\, ds \qquad \int \mathbf{F} \cdot d\mathbf{r} \qquad \int M\, dx + N\, dy$$

are all equivalent. They all describe a work-line integral.

Hints: Check to see if the vector field is a gradient field … consider using Green's Theorem.

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1. Find $\int \mathbf{F} \cdot \mathbf{T}\, ds$, where $\mathbf{F}(x, y) = \langle 3x, 2xy\rangle$ and *C* is a line segment from (1,2) to (4,7).
2. Find $\int \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle xy^2, 2x - y\rangle$ and *C* is a line segment from (6,6) to (3,0).
3. Find $\int M\, dx + N\, dy$, where $\mathbf{F}(x, y) = \langle x^2 + 2y, -x\rangle$ and *C* is a line segment from (-3,3) to (5,1).
4. Find $\int \mathbf{F} \cdot \mathbf{T}\, ds$, where $\mathbf{F}(x, y) = \langle 6y, x^2\rangle$ and *C* is the parabola $y = x^2$ from (1,1) to (4,16).
5. Find $\int M\, dx + N\, dy$, where $\mathbf{F}(x, y) = \langle 4, x^2 + y\rangle$ and *C* is the cubic $y = x^3$ from (-1,-1) to (2,8).
6. Find $\int \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle x^3 y, 2\rangle$ and *C* is the curve $y = 1 - x^2$ from (0,1) to (2,-3).
7. Find $\int \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle 3 - x, 1 + y\rangle$ and *C* is a circle of radius 2, centered at the origin, traversed counterclockwise.
8. Find $\int M\, dx + N\, dy$, where $\mathbf{F}(x, y) = \langle 2xy, 1 + 3x\rangle$ and *C* is a line from (1,2) to (5,1), then to (4,4).
9. Find $\int \mathbf{F} \cdot \mathbf{T}\, ds$, where $\mathbf{F}(x, y) = \langle 8xy, 4x^2\rangle$ and *C* is a sequence of line segments from (0,0) to (1,3) to (4,7) to (2,1).
10. Find $\int \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle 2x, 3y^2\rangle$ and *C* is a quarter circle of radius 4 centered at the origin from (4,0) to (0,4).
11. Find $\int M\, dx + N\, dy$, where $\mathbf{F}(x, y) = \langle y + 2xy^3, x + 3x^2 y^2\rangle$ and *C* is the curve $y = x^3 + x^2 - 2x + 1$ from (0,1) to (1,1).
12. Find $\int \mathbf{F} \cdot \mathbf{T}\, ds$, where $\mathbf{F}(x, y) = \langle \cos y, -x \sin y\rangle$ and *C* is a circle of radius 7, centered at (5,1).
13. Find $\int \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle 2x, 4xy\rangle$ and *C* is a triangle from (0,0) to (5,0) to (5,5) back to (0,0).
14. Find $\int M\, dx + N\, dy$, where $\mathbf{F}(x, y) = \langle 100y, 101x\rangle$ and *C* is a circle of radius 1, centered at $(\pi, \sqrt{2})$.
15. Find $\int \mathbf{F} \cdot \mathbf{T}\, ds$, where $\mathbf{F}(x, y) = \langle 2x, 80y^{10} + 3x\rangle$ and *C* is the parabola $y = 1 - x^2$ traversed from (1,0) to (-1,0), then back to (1,0) along the *x*-axis.