**Math 267**
**Flux Integrals Practice**

The form $\iint_C \mathbf{F}\cdot\mathbf{n}\,dS$ represents the flow (flux) of a vector field **F** through a surface *C*. Vector **n** is a normal to the surface. In the set-up, it is a unit vector, but in practice, that parts "cancels out" so that **n** does not have to remain a unit vector. However, it is important to get the right orientation of **n**, and not some inadvertent multiple of it. Please review my notes (typed and written) for more details in finding **n** and setting up a flux integral. Also, remember that the divergence theorem can be used when C is a surface enclosing a simply-connected subregion of $R^3$.

Rules: there are no answers here. You are encouraged to share your answers on Piazza!

Preliminary (surface area, surface integrals)

1. Find the area of $x+y+4z=12$ in the first octant.
2. Find the area of $z=\sqrt{x^2+y^2}$ such that $x^2+y^2\le 4$.
3. Find the area of $z=16-x^2-y^2$ that lies above the *xy*-plane.
4. Find the area of $z=\sqrt{25-x^2-y^2}$ that lies above the *xy*-plane.
5. Find $\iint_C (x+2y-z)\,ds$ where *C* is the plane $2x+y+3z=12$ in the first octant.

Flux integrals:

1. Find $\iint_C \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z)=\langle x,2x+y,2z\rangle$ and *C* is the portion of the plane $x+3y+2z=6$ in the first octant. Assume positive flow is in the positive *z* direction.
2. Find $\iint_C \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z)=\langle x,2x+y,2z\rangle$ and *C* is the portion of the plane $x+3y+2z=6$ in the first octant. Assume positive flow is in the positive *x* direction. (What do you notice happened with your answers in #1 and #2?)
3. Find $\iint_C \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z)=\langle x^2,1,2-y\rangle$ and *C* is the paraboloid $z=1-x^2-y^2$ above the *xy*-plane. Assume positive flow is in the positive *z* direction.

In problems 4-9, let solid *S* be a tetrahedron in $R^3$ that has vertices (0,0,0), (0,0,4), (0,3,0) and (2,0,0).

4. Find $\iint_C \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z)=\langle 2x,y,-z\rangle$ and *C* is the face of *S* with vertices (0,0,4), (0,3,0) and (2,0,0). Assume positive flow is in the positive *z* direction.
5. Find $\iint_C \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z)=\langle 2x,y,-z\rangle$ and *C* is the face of *S* on the *xy*-plane. Assume positive flow is in the negative *z*-direction.
6. Find $\iint_C \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z)=\langle 2x,y,-z\rangle$ and *C* is the face of *S* on the *yz*-plane. Assume positive flow is in the negative *x*-direction.
7. Find $\iint_C \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z)=\langle 2x,y,-z\rangle$ and *C* is the face of *S* on the *xz*-plane. Assume positive flow is in the negative *y*-direction.
8. Sum your results in Problems 4-7.
9. Now, use the divergence theorem to find $\iint_C \mathbf{F}\cdot\mathbf{n}\,dS$ where $F(x,y,z)=\langle 2x,y,-z\rangle$ and *C* is the surface of *S*, positive orientation away from the interior of *S*.

10. Find $\iint_C \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z)=\langle 2x,5z,-3y\rangle$ and *C* is a sphere with radius 5, centered at the origin.
11. Find $\iint_C \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z)=\langle 5z,100x,-7y\rangle$ and *C* is a tetrahedron with vertices (0,1,2), (3,4,5), (6,7,8), and (9,10,11). Hint: it's easy.
12. You are given that $\mathbf{F}(x,y,z)=\langle 9y^2,e^{2x}+z,-3\sqrt{y}\rangle$ and have determine the flux through three sides of a four sided object to be 19 through side 1, $\pi$ through side 2, and $\sqrt{17}$ through side 3. What is the flux through side 4?