Calculus-III Tangent Planes Practice Problems.

Answers are not included. You are encouraged to work together and post ideas and comments on Piazza.

Example: Find the equation of the tangent plane to $f(x,y) = x^4y - y^2$ at $x_0 = 2$ and $y_0 = 3$, and use the tangent plane to estimate the value of $f(2.1,2.95)$.

Solution: We need the full point: $z_0 = f(x_0, y_0) = (2)^4(3) - (3)^2 = 39$. Thus, the point of tangency is (2,3,39).

The general form of a tangent plane is $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$.

Find the partial derivatives and evaluate at $x_0 = 2$ and $y_0 = 3$:

$$\begin{matrix} f_x(x,y) = 4x^3y \\ f_y(x,y) = x^4 - 2y \end{matrix} \rightarrow \begin{matrix} f_x(2,3) = 4(2)^3(3) = 96 \\ f_y(2,3) = (2)^4 - 2(3) = 10 \end{matrix}$$

Now, assemble your plane:

$$z - 39 = 96(x - 2) + 10(y - 3).$$

Simplify by clearing parentheses: $z - 39 = 96x - 192 + 10y - 30$, then isolate *z*:

$$z = 96x + 10y - 183.$$

To estimate $f(2.1,2.95)$, use the plane:

$$\begin{aligned} z &= 96(2.1) + 10(2.95) - 183 \\ &= 201.6 + 29.5 - 183 \\ &= 48.1. \end{aligned}$$

The *actual* value of $f(2.1, 2.95)$ is $(2.1)^4(2.95) - (2.95)^2 = 48.669395$ ... . Note that the plane gave a very close approximation that uses easier arithmetic. This is also a good check of your work.

1. Find the equation of the tangent plane to $f(x,y) = x^2 + y^2$ at $x_0 = 3$ and $y_0 = 4$, and use the tangent plane to estimate the value of $f(3.1, 3.9)$.
2. Find the equation of the tangent plane to $f(x,y) = 2xy^2$ at $x_0 = 1$ and $y_0 = -2$, and use the tangent plane to estimate the value of $f(1.05, -1.9)$.
3. Find the equation of the tangent plane to $f(x,y) = x^3y - 2x$ at $x_0 = -1$ and $y_0 = 3$, and use the tangent plane to estimate the value of $f(-1.02,3.04)$.
4. Find the equation of the tangent plane to $f(x,y) = \sqrt{x - 2y}$ at $x_0 = 5$ and $y_0 = 1$, and use the tangent plane to estimate the value of $f(5.1,1.06)$.
5. Find the equation of the tangent plane to $f(x,y) = \ln(x^2 - y^3)$ at $x_0 = 4$ and $y_0 = 2$, and use the tangent plane to estimate the value of $f(4.2,1.9)$.
6. Find the equation of the tangent plane to $f(x,y) = \frac{6x}{y}$ at $x_0 = 1$ and $y_0 = 4$, and use the tangent plane to estimate the value of $f(1.04,3.98)$.

7. Find the equation of the tangent plane to $f(x,y) = \frac{x+y}{x-y}$ at $x_0 = -2$ and $y_0 = 5$, and use the tangent plane to estimate the value of $f(-2.1,4.9)$.
8. Find the equation of the tangent plane to $f(x,y) = e^{xy}$ at $x_0 = 1$ and $y_0 = 2$, and use the tangent plane to estimate the value of $f(1.1,1.93)$.
9. Find the equation of the tangent plane to $f(x,y) = x \sin y$ at $x_0 = 6$ and $y_0 = \frac{\pi}{2}$, and use the tangent plane to estimate the value of $f(6.1,1.5)$.
10. Find the equation of the tangent plane to $f(x,y) = x^3 - y^2$ at $x_0 = 8$ and $y_0 = 11$, and use the tangent plane to estimate the value of $f(8.2,11.2)$.