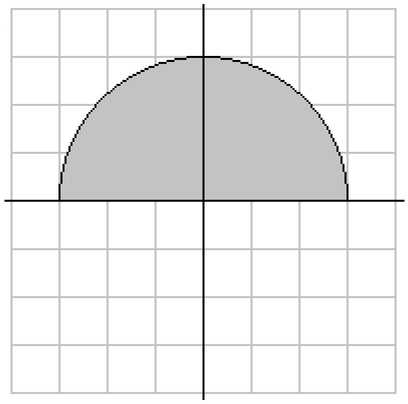
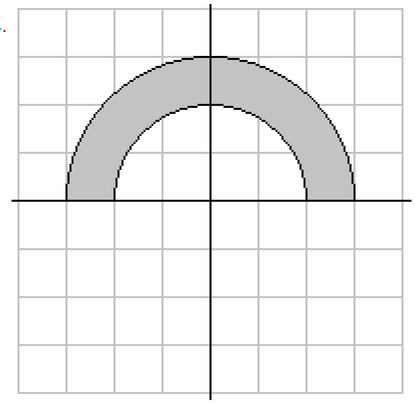
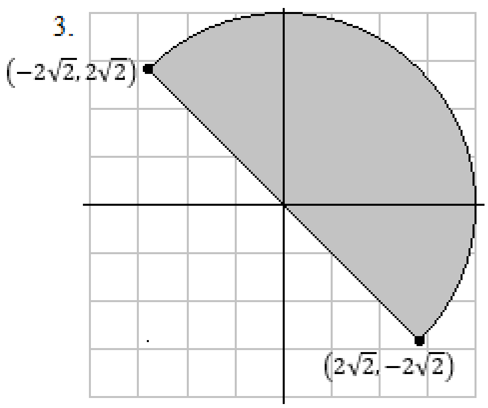
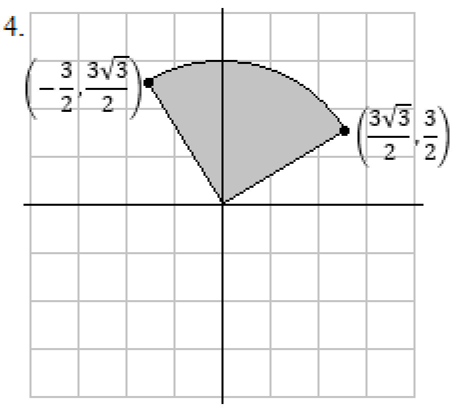
Bounds Practice – Polar, Cylindrical and Spherical

Describe these regions using polar bounds.

1.


2.


3.


4.


Translate these integrals into polar coordinates. Then find their exact answers. Hint: draw the regions.

5. $\displaystyle\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\, dy\, dx\,.$

6. $\displaystyle\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (1+2x^2+2y^2)\, dy\, dx.$

7. $\displaystyle\int_0^3 \int_{\sqrt{9-x^2}}^{\sqrt{25-x^2}} \frac{1}{4x^2+4y^2+1}\, dy\, dx + \int_3^5 \int_0^{\sqrt{25-x^2}} \frac{1}{4x^2+4y^2+1}\, dy\, dx.$

9. Find the volume contained within the paraboloid $z = 16 - x^2 - y^2$ that lies above the *xy*-plane.

10. Find the volume within the paraboloid $z = 25 - x^2 - y^2$ that lies in the first octant.

11. Find the volume within the paraboloid $z = 20 - x^2 - y^2$ that lies above the plane *z* = 11.

12. Find the volume of the paraboloid $z = 20 - x^2 - y^2$ that lies above the *xy*-plane but below the plane *z* = 11.

13. Find the volume within the paraboloids $z = x^2 + y^2$ and $z = 18 - x^2 - y^2$.

14. Find the volume within the cylinder $x^2 + y^2 = 36$ that lies above the *xy*-plane and below $x + z = 12$.

Evaluate these integrals.

15. $\iiint_R x^2 + y^2 + z^2 \; dV$, where *R* is a sphere centered at the origin of radius 3.

16. $\iiint_R \sqrt{x^2 + y^2 + z^2} \; dV$, where *R* is a hemisphere above the *xy*-plane with radis 2.

17. $\iiint_R (2x^2 + 2y^2 + 2z^2 + 1) \; dV$, where *R* is the space between spheres centered at the origin of radii 2 and 4, within the first octant.

18. $\int_0^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_0^{\sqrt{16-x^2-y^2}} x \, dz \, dy \, dx.$

19. A hemisphere of radius 6 centered at the origin and above the *xy*-plane has a conical sector removed such that the point (2,1,4) lies on the rim. Find the volume of the remaining solid.

20. Two spheres of radius 1 intersect one another such that each sphere passes through the other's center, creating a lens-shaped region. Find that region's volume.