Unconstrained Optimization

No answers are provided, so please share ideas and discussions on Piazza.

Find all critical points and classify them as minimum, maximum or saddle:

1. $f(x,y) = x^2 + y^2 - 4x + 8y - 3$
2. $f(x,y) = 3x^2 + 2y^2 + 6x + 10y - 1$
3. $g(x,y) = x^2 + xy + y^2 - 5x + 2y + 1$
4. $g(x,y) = x^3 + y^3 - 12x - 27y + 2$
5. $h(x,y) = \ln(x^2 + 5xy + y^2 - 4x - 9y)$
6. $h(x,y) = x^2 + 6xy + y^2 + x + 2y + 1$
7. $k(x,y) = x^4 + y^4$

Constrained Optimization

Find all critical points on the surface, subject to the given constraint. You may use substitution or Lagrange.

1. $f(x,y) = x^2 + y^2 + 3xy + 2x$, such that $x + 3y = 6$
2. $f(x,y) = xy$, such that $2x + 3y = 12$
3. $f(x,y) = 3x + 4y$, such that $x^2 + y^2 = 16$
4. $f(x,y) = x^2 + y^2$, such that $x^2 + y^2 + 4x + 6y = 16$
5. $f(x,y) = x^3 + y^3 - 3xy$, such that $2x - y = 4$
6. $f(x,y) = xy$, such that $y = x^2 - 4$
7. $f(x,y) = x + y + xy$, such that $x^2 + y^2 = 1$
8. Find the largest possible area of a square with one corner on the origin and its opposite corner on the line $4x + 7y = 28$.
9. Find the largest possible volume of a box with one corner on the origin and its opposite corner on the plane $5x + 2y + 6z = 30$.
10. Find the point on the plane $3x + 6y + 4z = 24$ that is closes to the origin.