Extreme Value Theorem (EVT)

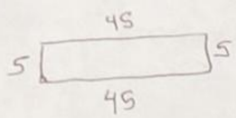
If $y=f(x)$ is a continuous function on a closed interval, then it is guaranteed to have an absolute min and max, possibly at an endpoint.

Ex: $y = x^2 - 6x + 1$, $\quad 1 \le x \le 6$

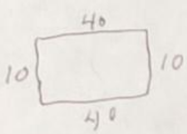
$y' = 2x - 6$ $\quad$ Dec: $(-\infty, 3)$
$2x - 6 = 0$ $\quad$ Inc: $(3, \infty)$
$x = 3$ $\quad$ $- \cup +$ MIN
$(3, -8)$ rel. min

check endpoints:
$(1, -4)$
$(6, 1)$ ★ Abs. MAX
$(3, -8)$ ★ Abs MIN

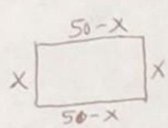
Ex: 100 feet of fencing will be used to create a rectangular enclosure. Find the dimensions that maximize the area.



Area $= 45 \cdot 5 = 225$ ft^2



Area $= 40 \cdot 10 = 400$ ft^2



left over: $100 - 2x$
split in half: $\frac{100-2x}{2} = \frac{\cancel{2}(50-x)}{\cancel{2}}$

Area $\Rightarrow$ $A(x) = x(50-x) = 50x - x^2$ $\qquad 0 \le x \le 50$

$A'(x) = 50 - 2x$
$50 - 2x = 0$
$-2x = -50$
$x = \frac{-50}{-2} = 25$
other side is 25

$\Big\}$ Area: 625 ft^2

Feb 24

A drug is administered into the bloodstream of a patient. Let $C(t) = 25te^{-0.04t}$ be the concentration, C, in nanograms per milliliters, t minutes after it was administered. What is the peak concentration of the drug, and when does it occur?

$$C'(t) = (25t)(-0.04e^{-0.04t}) + (e^{-0.04t})(25)$$
$$= -te^{-.04t} + 25e^{-.04t}$$
$$= e^{-.04t}(-t+25)$$

Set to 0: $e^{-.04t}(\underbrace{-t+25}) = 0$

$$-t+25=0$$
$$t=25$$

Concentration: $C(25) = 229.92$ ng/ml

(25, 229.92) Abs max.

When is the rate of change of the concentration decreasing the fastest?

It happens at the point of inflection.

Need the 2nd deriv:

$$C'(t) = e^{-.04t}(25-t)$$
$$C''(t) = (e^{-.04t})(-1) + (25-t)(-.04e^{-.04t})$$
$$= -e^{-.04t} - e^{-.04t} + 0.04te^{-.04t}$$
$$= -2e^{-.04t} + .04te^{-.04t}$$
$$= e^{-.04t}(-2+.04t)$$

set to 0:

$$e^{-.04t}(\underbrace{-2+.04t}) = 0$$
$$-2+.04t=0$$
$$.04t=2$$
$$t = \frac{2}{.04} = 50$$

At 50 mins, the concentration is decreasing the fastest (Point of Inflection)

Feb 24

The owner of a large hotel sells 120 rooms per night at a cost of \$100 per room. For every \$10 increase in price, the owner sells 5 less rooms. At what price should the owner sell each room for, and what would be the maximum revenue?

quantity ×	price	= revenue
120 room ×	\$100/room	= \$12,000
115	× \$110	= \$12650
110	× \$120	= \$13,200

$(120 - 5x)(100 + 10x) = R(x)$

FOIL

$R(x) = 12000 + 1200x - 500x - 50x^2$

$R(x) = 12000 + 700x - 50x^2$

$R'(x) = 700 - 100x$

$700 - 100x = 0$

$x = 7$ ☺

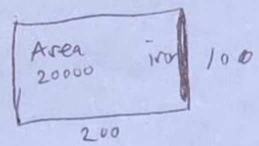
Set each room to \$170

sell : 85 rooms

$170 \times 85 =$ \$14,450 max rev.

Feb 24

A farmer is going to build a rectangular enclosure for his cows. One side will be iron fencing at \$15 per linear foot, the other three sides will be barbed wire at \$7 per linear foot. The area of the enclosure is to be 20,000 square feet. What dimensions will minimize the cost to ~~enclose~~ build the enclosure?

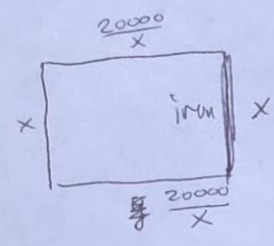


cost:

iron side $100 \times 15 = 1500$

barb sides: $200 \times 7 + 100 \times 7 + 200 \times 7 = 3500$

\$5000



cost:

iron: $15x +$

barb: $\left(\frac{20000}{x}\right)7 + 7x + \left(\frac{20000}{x}\right)7$

$$C(x) = 22x + \frac{280000}{x}$$

Area: $xy = 20000$

$y = \frac{20000}{x}$

$$C'(x) = 22 - \frac{280000}{x^2}$$

$$22 - \frac{280000}{x^2} = 0$$

$$22 = \frac{280000}{x^2}$$

$$22x^2 = 280000$$

$$x^2 = \frac{280000}{22}$$

$$x = \sqrt{\frac{280000}{22}}$$

$= 112.82$ x short side

long side: $\frac{20000}{112.82} = 177.27$

Cost: \$4963.87

Feb 24

Find any min/max on $f(x) = e^{3x} - e^{x}$
and also locate its point of inflection.

$$f'(x) = 3e^{3x} - e^{x}$$

set to 0:

$$3e^{3x} - e^{x} = 0$$

$$e^{x}(3e^{2x} - 1) = 0$$

No sol. $\quad 3e^{2x} - 1 = 0$

$$3e^{2x} = 1$$

$$e^{2x} = \frac{1}{3}$$

ln both sides: $\ln e^{2x} = \ln\left(\frac{1}{3}\right)$

$$2x = \ln\left(\frac{1}{3}\right)$$

$x = \frac{1}{2}\ln\left(\frac{1}{3}\right)$ crit. val.

$$\approx -0.549$$

to get the y-value, find $f(-0.549) \approx -0.385$

$(-0.549, -0.385)$ critical pt
rel. MIN
Abs.

inc: $(-0.549, \infty)$
dec: $(-\infty, -0.549)$

$- \cup +$
mAN

point of inflection. Find $f''(x)$.

Recall: $f'(x) = 3e^{3x} - e^{x}$

$$f''(x) = 9e^{3x} - e^{x}$$

set it to 0: $9e^{3x} - e^{x} = 0$

$$e^{x}(9e^{2x} - 1) = 0$$

$$9e^{2x} - 1 = 0$$

$$9e^{2x} = 1$$

$$e^{2x} = \frac{1}{9}$$

$$2x = \ln\frac{1}{9}$$

$$x = \frac{1}{2}\ln\frac{1}{9} \approx -1.0986$$

Feb 26