$y = x^2 + 3x + 1$

$y = (x^2 + 5x + 2)^3$

$f(x) = x^3, \quad g(x) = 1 + x^2$

$(f \circ g)(x) = f(g(x)) \rightarrow f(1+x^2) = (1+x^2)^3 \rightarrow (f \circ g)' = 3(1+x^2)^2(2x$

$(g \circ f)(x) = g(f(x)) = g(x^3) = 1 + (x^3)^2 = 1 + x^6$

deriv: $(g \circ f)' = 6x^5$

---

$f(x) = (x^3 + 2x + 1)^4$

$f' = 4(x^3 + 2x + 1)^3 (3x^2 + 2)$

$g(x) = \sqrt{x^2 + x}$

$g' = \dfrac{2x + 1}{2\sqrt{x^2 + x}}$

$y = (x^5 + 2x^3 + x)^{10}$

$y' = 10(x^5 + 2x^3 + x)^9 (5x^4 + 6x^2 + 1)$

$g(x) = \sqrt[3]{3x^5 + 2}$

$g' = \dfrac{15x^4}{3\sqrt[3]{(3x^5 + 2)^2}}$

$y = (\sqrt{x} + x)^7$

$y' = 7(\sqrt{x} + x)^6 \left(\dfrac{1}{2\sqrt{x}} + 1\right)$

$g(x) = \sqrt[4]{1 + 2x} \quad g' = \dfrac{2}{4\sqrt[4]{(1+2x)^3}}$

$y = (3x^5 + 2)^{\frac{1}{3}}$

$y' = \frac{1}{3}(3x^5 + 2)^{-\frac{2}{3}} (15x^4)$

$= \dfrac{15x^4}{3\sqrt[3]{(3x^5+2)^2}}$

$g = (1 + 2x)^{\frac{1}{4}}$

$g' = \frac{1}{4}(1 + 2x)^{-\frac{3}{4}} (2)$

$= \dfrac{2}{4\sqrt[4]{(1+2x)^3}}$

Feb3

$$y = 5(x^3+7x+1)^2 \qquad\qquad y = \sqrt{1+\sqrt{x}}$$

$$y' = 10(x^3+7x+1)^1(3x^2+7) \qquad\qquad y' = \frac{\left(\frac{1}{2\sqrt{x}}\right)}{2\sqrt{1+\sqrt{x}}}$$

$$f(x) = (3x-5)^4(7-x)^{10}$$

$$f' = \left(\,(3x-5)^4\,\right)\left(10\underbrace{(7-x)^9(-1)}_{\text{chain.}}\right) + \left(\,(7-x)^{10}\,\right)\left(4\underbrace{(3x-5)^3(3)}_{\text{chain.}}\right)$$

$$f(x) = \frac{2x^2-1}{(3x^4+2)^2}$$

$$f' = \frac{\left((3x^4+2)^2\right)\left(\,4x\,\right) - \left(2x^2-1\right)\left(2\underbrace{(3x^4+2)^1(12x^3)}_{\text{CHAIN}}\right)}{\left((3x^4+2)^2\,\right)^2}$$

## Practice

Find the derivatives:

1. $y = (2x^2+3)(5x-2)$ $\qquad y' = (2x^2+3)(\ 5\ ) + (5x-2)(\ 4x\ )$

$= 10x^2+15 + 20x^2-8x$

$= 30x^2-8x+15$

2. $y = (\sqrt{x} + \sqrt[3]{x})(x^2+5x+1)$ $\qquad y' = (\sqrt{x}+\sqrt[3]{x})(2x+5) + (x^2+5x+1)\left(\frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}}\right)$

$x^{\frac{1}{3}} \Rightarrow \frac{1}{3}x^{-\frac{2}{3}}$

3. $y = \dfrac{4x+3}{7x+2} \rightarrow y' = \dfrac{\overset{28x+8}{(7x+2)(\ 4\ )} - \overset{-(28x+21)}{(4x+3)(\ 7\ )}}{(7x+2)^2} = -\dfrac{13}{(7x+2)^2}$

$(x \neq -\frac{2}{7})$

$\rightarrow x^{\frac{1}{4}} \rightarrow y' = \frac{1}{4}x^{-\frac{3}{4}}$

4. $y = \dfrac{\sqrt[4]{x}}{\sqrt[3]{x}+1} \rightarrow y' = \dfrac{(\sqrt[3]{x}+1)\left(\frac{1}{4\sqrt[4]{x^3}}\right) - (\sqrt[4]{x})\left(\frac{1}{3\sqrt[3]{x^2}}\right)}{(\sqrt[3]{x}+1)^2}$

5. $y = \dfrac{x^2+x-12}{x-3} = \dfrac{\cancel{(x-3)}(x+4)}{\cancel{(x-3)}} \rightarrow y' = 1, \quad x \neq 3$

2-5-26 p.1

## Chain rule

Find these derivatives

1. $y = (x^2+5)^6 \rightarrow y' = 6(x^2+5)^5(2x) = 12x(x^2+5)^5$

2. $y = \sqrt{x^2+7} \rightarrow y' = \dfrac{2x}{2\sqrt{x^2+7}}$ CH. $= \dfrac{x}{\sqrt{x^2+7}}$

3. $y = \sqrt[3]{x^3+3x} \rightarrow y' = \dfrac{3x^2+3}{3\sqrt[3]{(x^3+3x)^2}} = \dfrac{x^2+1}{\sqrt[3]{(x^3+3x)^2}}$

$y = x^{\frac{1}{3}}$

$y' = \frac{1}{3}x^{-\frac{2}{3}}$

$y' = \dfrac{1}{3\sqrt[3]{x^2}}$

4. $y = \dfrac{1}{(5x^2+3x)} \rightarrow y' = -\dfrac{10x+3}{(5x^2+3x)^2}$

$y = \frac{1}{x} = x^{-1}$

$y' = -1x^{-2} = -$

5. $y = (2x+1)^3(x^2+5)^4 \rightarrow y' = (2x+1)^3(4(x^2+5)^3(2x)) + (x^2+5)^4(3(2x+1)^2 \cdot 2)$

6. $y = \dfrac{(x^2+6)^4}{(7x^3+5)^3} \rightarrow y' = \dfrac{(7x^3+5)^3(4(x^2+6)^3(2x)) - (x^2+6)^4(3(7x^3+5)^2(21x^2))}{(7x^3+5)^6}$ ← "cube squared"

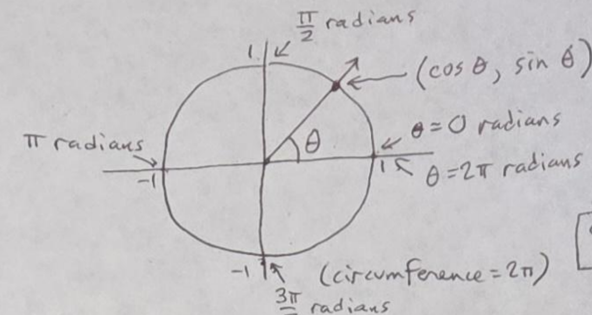
Feb 5

$y = x^2 \sin(6x^3) \rightarrow y' = (x^2)(18x^2\cos(6x^3)) + (\sin(6x^3))(2x)$ CH

$y = \dfrac{2x^4}{\cos(x^2)} \rightarrow y' = \dfrac{(\cos x^2)(8x^3) - (2x^4)(-2x\sin x^2)}{(\cos(x^2))^2}$

# TRIG Study of angles.

shapes we use: unit circle circle, rad = 1, centered at origin.

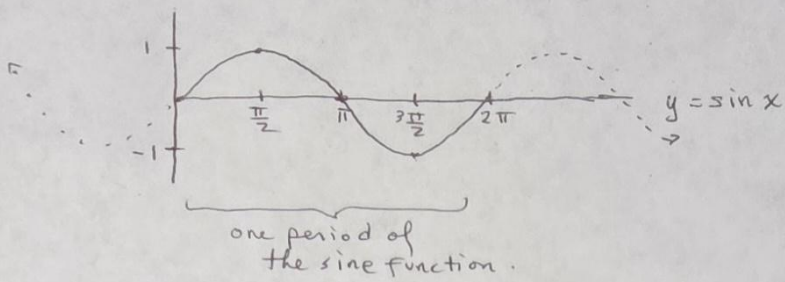


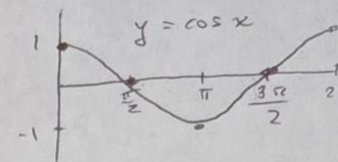

x coord: $\cos\theta$

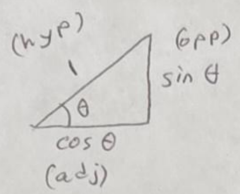
y coord: $\sin\theta$

slope of the ray: $\tan\theta$

[ $\cot\theta = \frac{1}{\tan\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\csc\theta = \frac{1}{\sin}$




→ pythagorean Identity: $\boxed{(\cos\theta)^2 + (\sin\theta)^2 = 1}$

→ SOH CAH TOA

$\sin = \frac{opp}{hyp}$ $\cos = \frac{ADJ}{HYP}$ $TAN = \frac{OPP}{ADJ}$ $\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$

Let's find derivative of $f(x) = \sin x$

sum Identity

$$f'(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} = \lim_{h\to 0} \frac{\sin x \cosh + \cos x \sinh - \sin x}{h}$$

$$= \lim_{h\to 0} \sin x \underbrace{\left[\frac{\cos h - 1}{h}\right]}_{\to 0} + \cos x \underbrace{\left[\frac{\sin h}{h}\right]}_{\to 1}$$

$$= \cos x$$

$$\boxed{\begin{array}{lll} \text{If } f(x) = \sin x & \text{then} & f'(x) = \cos x \\ \text{If } f(x) = \cos x & \text{then} & f'(x) = -\sin x \\ \text{If } f(x) = \tan x & \text{then} & f'(x) = \sec^2 x \end{array}}$$

Know these three forever!

$y = \sin(2x) \rightarrow y' = \cos(2x)(\overset{CH}{2}) = 2\cos(2x)$

$y = \cos(3x^2) \rightarrow y' = -\underset{CH}{6x}\sin(3x^2)$

$y = \tan(x^2+3x) \rightarrow y' = \underset{CH.}{(2x+3)}\sec^2(x^2+3x)$