Notes Week 3 (MAT251)

$$y = f(x)\cdot g(x)$$

$$y' = f(x)\cdot g'(x) + f'(x)\cdot g(x)$$

$$= \text{First} \times \text{Deriv. } 2^{nd} + 2^{nd} \times \text{Deriv } 1^{st}$$

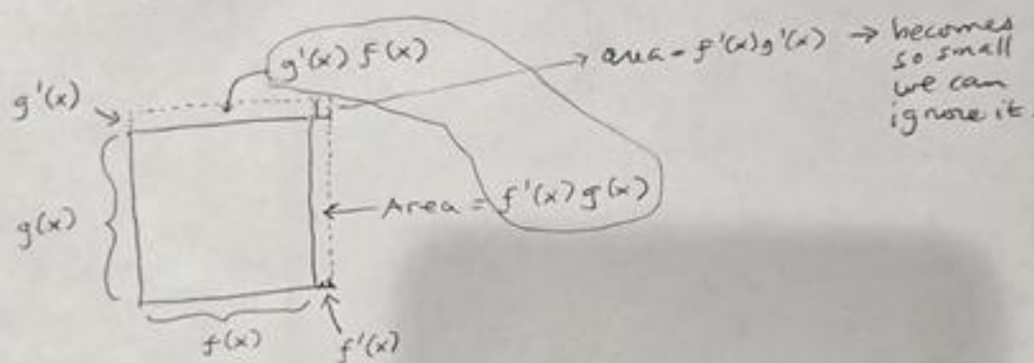
proof:

$$\lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$\lim_{h\to 0} \frac{f(x+h)g(x+h) \overbrace{- f(x+h)g(x) + f(x+h)g(x)}^{\text{Add in, take out.}} - f(x)g(x)}{h}$$

$$\lim_{h\to 0} f(x+h)\underbrace{\left[\frac{g(x+h)-g(x)}{h}\right]} + \lim_{h\to 0} g(x)\underbrace{\left[\frac{f(x+h)-f(x)}{h}\right]}$$

$$f(x)\cdot g'(x) + g(x)\cdot f'(x)$$



$$y = \frac{f(x)}{g(x)} \begin{matrix}\text{TOP}\\ \text{BOTTOM}\end{matrix} \qquad \boxed{y' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}}$$

$$\frac{\text{LOW} * \text{D-HI} - \text{HI} * \text{D-LOW}}{(\text{Below})^2}$$

Low D HI
MINUS
HI D-Low
All over the square
of what's below

$$\lim_{h \to 0} \left[\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} \right] = \frac{\frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)g(x)}}{h}$$

"adding 0"

$$= \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h\,g(x+h)g(x)}$$

$$= \frac{g(x)[f(x+h) - f(x)] + f(x)[g(x) - g(x+h)]}{h\,g(x+h)g(x)}$$

$$= \frac{1}{g(x+h)g(x)} \left[g(x) \left[\frac{f(x+h) - f(x)}{h} \right] - f(x) \left[\frac{g(x+h) - g(x)}{h} \right] \right]$$

let $\lim_{h \to 0}$

$$\frac{1}{(g(x))^2} \left[g(x) f'(x) - f(x) g'(x) \right]$$

$$\frac{g(x) f'(x) - f(x) g'(x)}{(g(x))^2}$$

$$y = \frac{x^2 - 3x}{x - 1}$$

$$y' = \frac{\overset{\text{Low}}{(x-1)}\overset{\text{D HI}}{(2x-3)} - \overset{\text{HI}}{(x^2-3x)}\overset{\text{D-LO}}{(1)}}{\underset{\text{Lo}}{(x-1)^2}}$$

$$y=(x^4-2x^3-7)(3x^2-5x)$$

$$y' = \underbrace{(x^4-2x^3-7)}_{1^{st}}\underbrace{(6x-5)}_{D-2^{nd}} + \underbrace{(3x^2-5x)}_{2^{nd}}\underbrace{(4x^3-6x^2)}_{D-1^{st}}$$

Done!

$$y=(2x+1)(3x+5)$$ → this could be expanded first, then differentiated.

$$y=(\sqrt{x}+1)(\underbrace{\sqrt[5]{x}}_{=x^{1/5}}-x)$$

$$y' = \underbrace{(\sqrt{x}+1)}_{1}\underbrace{\left(\tfrac{1}{5}x^{-\frac{4}{5}}-1\right)}_{D2} + \underbrace{(\sqrt[5]{x}-x)}_{2}\underbrace{\left(\frac{1}{2\sqrt{x}}\right)}_{D1}$$

$$(\sqrt{x}+1)\left(\frac{1}{5\sqrt[5]{x^4}}-1\right) + (\sqrt[5]{x}-x)\left(\frac{1}{2\sqrt{x}}\right)$$

$$y = \frac{x^2+4x+3}{x+1}$$ → simplify first: $\frac{\cancel{(x+1)}(x+3)}{\cancel{(x+1)}} \rightarrow y = x+3$, $x \neq -1$

$$y' = \frac{(x+1)(2x+4) - (x^2+4x+3)(1)}{(x+1)^2}$$

$$= \frac{2x^2+6x+4-x^2-4x-3}{(x+1)^2}$$

$$= \frac{x^2+2x+1}{(x+1)^2}$$

$$= \frac{(x+1)^2}{(x+1)^2}$$

$$= 1$$

huh?

If possible, simplify First!