Tue, 1-20-26

$(\cancel{x-4})(x+12)$

$$\lim_{x\to 4} \frac{x^2+8x-48}{\cancel{x-4}} = 16$$

not continuous at $x=4$
because $f(4)$ DNE.

$\boxed{\lim_{x\to a} f(x) = f(a)}$

4 is not allowed.
Calculator says "ERROR".
ERROR does NOT mean "DNE".

Options: ① factor, cancel.
② Look at Table in calculator
③ Graph it too.

$$\lim_{x\to 5} \frac{x^2+8x-48}{x-4}$$

just plug in 5: 17

lim exists, = 17.
$f(5) = 17$
$\lim_{x\to 5} f(x) = f(5)$

we have continuity at $x=5$.

$$\lim_{x\to\infty} \frac{x^3+7x}{6x^3+9x^2+2} = \frac{1}{6}$$

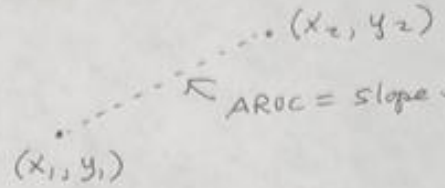
coeffs of leading terms

graph is levelling off at $y=\frac{1}{6}$ as $x\to\infty$ ($\pm$)

1-20-26

Average Rate of Change "AROC"

just a slope



AROC = slope of the line $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\Delta y}{\Delta x}$

Δ "Delta" means change

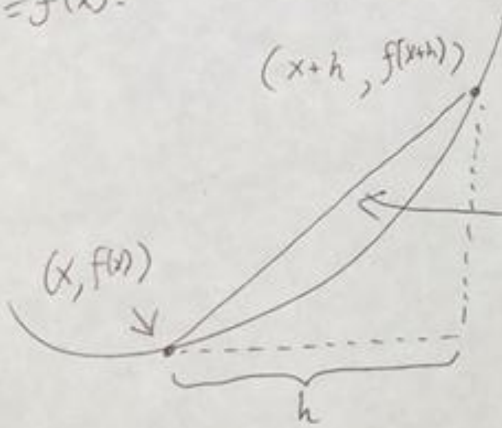
You go on a bike ride. After 1 hr, you're 5 miles from home.
After 3 hrs, you're 19 miles from home.

$\Rightarrow$ (1,5) and (3,19)

$\Rightarrow$ slope: $\dfrac{\Delta y}{\Delta x} = \dfrac{19-5}{3-1} = \dfrac{14}{2} = 7 \ \dfrac{\text{miles}}{\text{hr}}$ } this is a rate

over this 2 hr period, you averaged 7 $\dfrac{\text{miles}}{\text{hr}}$.

$y = f(x)$:



slope: $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x+h) - f(x)}{\cancel{x} + h - \cancel{x}}$

$$\frac{f(x+h) - f(x)}{h}$$

Difference Quotient

$$\frac{f(x+h)-f(x)}{h}$$

$f(x)=x^2$

b) $x=5, \quad h=.1$

generate 2 pts: $f(5)=25$, $f(5.1)=26.01$

$$\frac{\Delta y}{\Delta x}=\frac{26.01-25}{5.1-5}=\frac{1.01}{.1}=10.1$$

slope of the secant line between $x=5$, $x=5.1$

do it with $h=.01$

$$\frac{f(5.01)-f(5)}{.01}=\frac{25.1001-25}{.01}=\frac{.1001}{.01}=10.01$$

as $h \to 0$

slopes are $\to 10$

This was a limit!

this is an instantaneous rate of change.

IROC

Thu, 1-22-26

Average rate of change

slope of a line between 2 points on a curve

calculated using slope formula

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\boxed{\text{difference quotient} \quad \frac{f(x+h)-f(x)}{h}}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^3 - x^3}{h}$$

expand this:

$$(x+h)(x+h)(x+h)$$

$$x^2 + xh + xh + h^2$$

$$= (x^2 + 2xh + h^2)(x+h)$$

$$= x^3 + 2x^2h + xh^2 + hx^2 + 2xh^2 + h^3$$

$$= x^3 + 3x^2h + 3xh^2 + h^3$$

$$f(x) = x^2$$

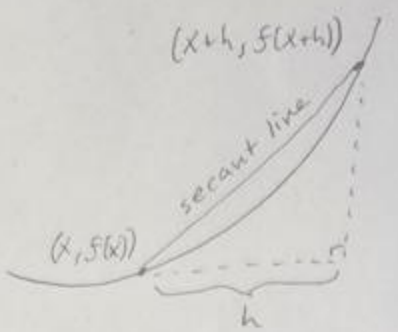
$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

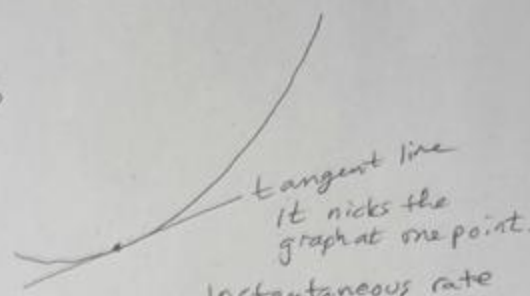
$$= 2x + h, \quad h \neq 0$$

If we let $h \to 0$:

$$\lim_{h \to 0} (2x+h) = 2x$$



if $h \to 0$



Instantaneous rate of change "IROC"

Nees calculus (derivative) to answer this

We've seen this so far:

$f(x) = x^3$, the derivative is $f'(x) = 3x^2$ "prime"

$f(x) = x^2$, the derivative is $f'(x) = 2x$

$f(x) = \frac{3}{x}$, the derivative is $f'(x) = -\frac{3}{x^2}$

$\boxed{\text{THE DERIVATIVE is A Function that gives the slope of a tangent line at any point on the graph.}}$

graph $y = x^2$

$f(x) = 3x \rightarrow f'(x) = 3$ (lines have constant slope)

$f(x) = 2 \rightarrow f'(x) = 0$ deriv. of a constant is 0.

Power rules:

$f(x) = x^3 \rightarrow f'(x) = 3x^2$

$f(x) = x^2 \rightarrow f'(x) = 2x$

$f(x) = 3x \rightarrow f'(x) = 3$

$f(x) = 2 \rightarrow f'(x) = 0$

$$f(x) = x^n \rightarrow f'(x) = nx^{n-1}$$

$f(x) = x^4 \rightarrow f'(x) = 4x^3$

$f(x) = x^{25} \rightarrow f'(x) = 25x^{24}$

$f(x) = \frac{3}{x} = 3x^{-1} \rightarrow f'(x) = -3x^{-2} = -\frac{3}{x^2}$

$f(x) = \frac{2}{7}x^{3/4} \rightarrow f'(x) = \frac{2}{7}\left(\frac{3}{4}x^{-\frac{1}{4}}\right) = \frac{3}{14\sqrt[4]{x}}$

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$