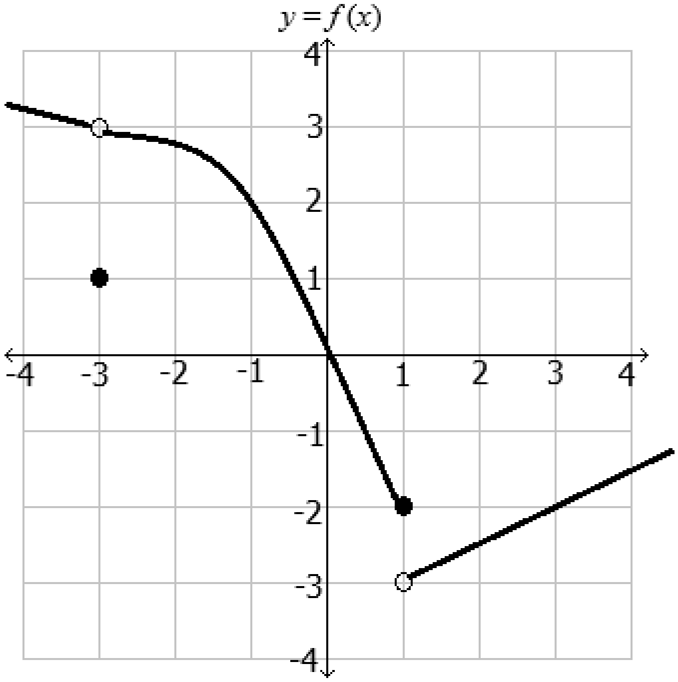
MAT251 – Test 1 – February 12, 2026 – Surgent **Key**

There were some slight differences between the two forms..

The graph below is used for questions 1-6. (2 points each)



1. Find $\lim_{x\to -3} f(x) = 3$

2. Form A: Find $\lim_{x\to -1} f(x) = 2$
   Form B: Find $\lim_{x\to 3} f(x) = -2$

3. Find $\lim_{x\to 1^-} f(x) = -2$

4. Find $\lim_{x\to 1^+} f(x) = -3$

5. Find $\lim_{x\to 1} f(x) = \text{DNE}$

6. (Form A) Why is f discontinuous at $x = -3$?
   (Use the definition of continuity to explain).

   $$\lim_{x\to -3} f(x) \neq f(3) \qquad (3 \neq 1)$$

   (Form B) Why is f discontinuous at $x = 1$?
   (Use the definition of continuity to explain)

   Limit at $x = 1$ does not exist.

Use any method to find these limits. (5 pts each)

7. $\lim_{x\to 2}(x^2 + 6x + 1) = (2)^2 + 6(2) + 1 = 17$

8. $\lim_{x\to 3} \frac{x^2+x-12}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+4)}{x-3}$

   $= \lim_{x\to 3} (x+4) = (3) + 4 = 7$

9. $\lim_{x\to 4} \frac{x^2+x+1}{x+1} = \frac{(4)^2+(4)+1}{(4)+1} = \frac{21}{5}$

10. Form A: $\lim_{x\to\infty} \frac{3x^4+7x^2-x}{6x^4+8x^3+10x} = \frac{3}{6}$ or $\frac{1}{2}$
    Form B: $\lim_{x\to\infty} \frac{6x^4+7x^2-x}{3x^4+8x^3+10x} = \frac{6}{3} = 2$

11. A cannonball is shot off a cliff 100 feet high. The height (in feet) of the cannonball above the ground is given by $f(t) = -16t^2 + 30t + 100$, where t is in seconds.

a. Using a difference quotient, fill in the following table for the indicated values of t and h. Show all decimal places (don't use TABLE in your calculator). Include units with your answers. (3 pts each)

At $t = 2$ and $h = 0.1$.	$\frac{f(2.1)-f(2)}{0.1} = \frac{92.44-96}{0.1} = -\frac{3.56}{0.1} = -35.6\frac{\text{f}}{\text{s}}$
At $t = 2$ and $h = 0.01$.	$\frac{f(2.01)-f(2)}{0.01} = \frac{95.6584-96}{0.01} = -\frac{0.3416}{0.01} = -34.16\frac{\text{f}}{\text{s}}$
At $t = 2$ and $h = 0.001$.	$\frac{f(2.01)-f(2)}{0.001} = \frac{95.965984-96}{0.001} = -\frac{0.034016}{0.001} = -34.016\frac{\text{f}}{\text{s}}$

Form B: The answers were -3.6 f/s, -2.16 f/s, -2.016 f/s

b. Using your answers in the table above, infer the instantaneous rate of change of the cannonball at $t = 2$ seconds. Include units. (3 pts)

Form A: Instantaneous rate of change at $t = 2$ seconds is -34 feet/sec
Form B: Instantaneous rate of change at $t = 1$ second is -2 feet/sec

Find the derivative of the following functions. Use derivative notation to identify the derivative. (5 pts each)

12. $f(x) = 5x^3 + \frac{7}{2}x^2 - 4x + 25$

$$f'(x) = 15x^2 + 7x - 4$$

13. $g(x) = \sqrt{x^3 + 2x + 1}$

Form A: $g'(x) = \frac{3x^2 + 2}{2\sqrt{x^3 + 2x + 1}}$

Form B: $g'(x) = \frac{2x + 3}{2\sqrt{x^2 + 3x + 1}}$

14. $k(t) = \frac{5}{8x^3}$

Form A: Note that $\frac{5}{8x^3} = \frac{5}{8}x^{-3}$

$$k'(t) = -\frac{15}{8}x^{-4} = -\frac{15}{8x^4}$$

Form B: $k'(t) = -\frac{32}{5x^5}$

15. $m(x) = 10x^9 + 6x^{1.5} + 3^2$

$$m'(x) = 90x^8 + 9x^{0.5}$$

Note that 3^2 is constant so its derivative is 0.

Questions 16-19 are 8 pts each; Question 20 is 4 points.

16. What is the slope of the tangent line of $f(x) = -2x^3 + 6x^2 - 5$ at $x = 4$?

Form A: $f'(x) = -6x^2 + 12x,$ so the slope $= f'(4) = -6(4)^2 + 12(4) = -48$
Form B: slope $= f'(3) = -6(3)^2 + 12(3) = -18$

17. Find the derivative of $f(x) = (2x+1)^3(x^2-6x)^4$. Leave answer "spread out", no simplifying needed.

$$f'(x) = (2x+1)^2\big(4(x^2-6x)^3(2x-6)\big) + (x^2-6x)^4\big(3(2x+1)^2(2)\big)$$

18. Find the derivative of $f(x) = \frac{5x-2}{(x^2+7x+1)^3}$. Leave answer "spread out".

$$f'(x) = \frac{(x^2+7x+1)^3(5) - (5x-2)\big(3(x^2+7x-1)^2(2x+7)\big)}{((x^2+7x+1)^3)^2}$$

19. Find the derivative of $f(x) = \frac{x^2+2x-35}{x-5}$. Simplify your answer and state any restrictions.

Simplify first:

$$f(x) = \frac{(x-5)(x+7)}{x-5} = x+7, \qquad \text{so } f'(x) = 1, x \neq 5.$$

20. Very short answers.

a. Form A: True or False: the limit of $f(x)$ as $x \to a$ tells us the behavior of f at $x = a$. False
Form B: True or False: the limit of $f(x)$ as $x \to a$ tells us the behavior of f near $x = a$. True

b. Fill in the blank: The derivative of a function gives the <u>instantaneous</u> rate of change at a given point.