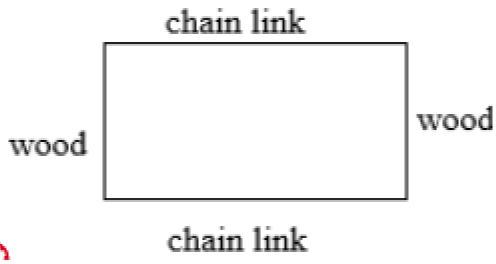
1. A medication is ingested by a patient. Let $C(t) = 30te^{-0.05t}$ be the concentration of the medication in ng/ml in the patient's bloodstream t minutes after ingestion. Using the derivative, find the maximum point giving the time and concentration when the medication is at its most concentrated. Show all steps and include a sketch of the graph with the maximum identified. (4 pts)

2. A mathematician is going to build a rectangular fence enclosing an area of 200 square meters. Two opposite sides will be wooden fencing costing $25 per meter to install (see image), and the other two opposite sides will be chain link costing $10 per meter to install. Set up a cost function and use the derivative to find the dimensions that minimize the entire cost to install the fencing. State the minimum cost, too. (6 pts)



1. Differentiate:

$$C'(t) = (30t)(-0.05e^{-0.05t}) + (30)(e^{-0.05t}) = -1.5te^{-0.05t} + 30e^{-0.05t}$$

Set to 0:

$$-1.5te^{-0.05t} + 30e^{-0.05t} = 0 \quad \rightarrow \quad e^{-0.05t}(-1.5t + 30) = 0$$

Solve for t:

$$-1.5t + 30 = 0 \quad \rightarrow \quad t = \frac{30}{1.5} = 20.$$

Use any test to show that $(20, C(20))$ is a relative max.
Thus, at $t = 20$ minutes, the max concentration is 220.728 ng/ml.

2. Let x = wood side and y = chain link side. Note that $xy = 200$ so that $y = \frac{200}{x}$.

The cost is

$$C = 25x + 25x + 10y + 10y$$

Replace y with $\frac{200}{x}$:

$$C = 25x + 25x + 10\left(\frac{200}{x}\right) + 10\left(\frac{200}{x}\right)$$

Simplify:

$$C(x) = 50x + \frac{4000}{x}.$$

Differentiate:

$$C'(x) = 50 - \frac{4000}{x^2}.$$

Set to 0 and solve:

$$50 - \frac{4000}{x^2} = 0 \quad \rightarrow \quad 50 = \frac{4000}{x^2} \quad \rightarrow \quad 50x^2 = 4000 \quad \rightarrow \quad x^2 = \frac{4000}{50}.$$

Note that 4000/50 = 80. Take square root:

$$x = \sqrt{80} \approx 8.94\, m.$$

Thus,

$$y = \frac{200}{8.94} = 22.36\, m.$$

The minimal cost will be

$$C(8.94) = 50(8.94) + \frac{4000}{8.94} = \$894.43.$$