MAT 267 – Exam 3 – Surgent –November 16, 2023 KEY Name:_____

PART I: Multiple Choice. Write the letter of your choice in the space provided at right. (6 pts each)

1. The value of $\int_0^1 \int_0^2 \int_0^3 (2x + 4y)\, dz\, dy\, dx$ is

Ans(1) B

A. 10 **B.** 30 **C.** 18 **D.** 2

2. Consider the triple integral $\iiint_E (x^2 + y^2 + z^2)\, dV$, where E is the solid sphere $x^2 + y^2 + z^2 \leq 1$ confined to the first octant (all variables are non-negative). In spherical coordinates, the integral is

Ans(2) D

A. $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho^2 \sin\phi \; d\rho\, d\theta\, d\phi$ **B.** $\int_0^{2\pi} \int_0^{\pi} \int_0^1 \rho^4 \sin\phi \; d\rho\, d\theta\, d\phi$

C. $\int_0^{2\pi} \int_0^{\pi} \int_0^1 \rho^2 \sin\phi \; d\rho\, d\theta\, d\phi$ **D.** $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho^4 \sin\phi \; d\rho\, d\theta\, d\phi$

$x^2 + y^2 + z^2 = \rho^2$, and Jacobian is $\rho^2 \sin\phi$

3. Which vector field below is conservative?

Ans(3) C

A. $\mathbf{F}(x, y) = \langle -y^2, x \rangle$ **B.** $\mathbf{F}(x, y) = \langle x^3 y, xy^3 + 1 \rangle$

C. $\mathbf{F}(x, y) = \langle x - 4y, y - 4x \rangle$ **D.** $\mathbf{F}(x, y) = \langle 2y + 1, 3 \rangle$

Check $M_y = N_x$

4. Evaluate $\int_C x\, ds$ where C is the arc of $x^2 + y^2 = 9$ from (3,0) to (0,3).

Ans(4) A

A. 9 **B.** -9 **C.** 3 **D.** -3

$r(t) = \langle 3\cos t, 3\sin t \rangle, 0 \leq t \leq \frac{\pi}{2}, ds = |r'(t)|dt = 3\, dt$

$\int_C x\, ds = \int_0^{\pi/2} 3\cos t \; (3\, dt) = [9\sin t]_0^{\pi/2} = 9(1 - 0) = 9$

5. The work performed by the vector field $\mathbf{F}(x, y) = \langle y^2, 2x + y \rangle$ on a particle moving along the path from (0,0) to (1,0) to (1,4) back to (0,0) is given by which integral?

Ans(5) D

A. $\int_0^1 \int_0^{4x} (2y - 2)\, dy\, dx$ **B.** $\int_0^1 \int_0^{4x} (1)\, dy\, dx$ **C.** $\int_0^1 \int_0^{4x} (-1)\, dy\, dx$ **D.** $\int_0^1 \int_0^{4x} (2 - 2y)\, dy\, dx$

Not conservative, C is a loop oriented ccw. $N_x - M_y = 2 - 2x$.

6. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle 3x^2, 2y \rangle$ and C is the path from (0,0) to (4,3) to (7,5) to (1,2).

Ans(6) C

A. 0 **B.** 73 **C.** 5 **D.** 68

Conservative, $f(x,y) = x^3 + y^2$. Evaluate at endpoints: $[x^3 + y^2]_{(0,0)}^{(1,2)} = 1^3 + 2^2 - 0 = 5$.

7. Which vector field has no curl?

Ans(7) A or D

A. $\mathbf{F}(x,y) = \langle x - 4y, y - 4x \rangle$ **B.** $\mathbf{F}(x,y) = \langle 2y + 1, 3 \rangle$
C. $\mathbf{F}(x,y) = \langle -y^2, x \rangle$ **D.** $\mathbf{F}(x,y) = \langle xy^2, x^2y + 1 \rangle$

Conservative fields have no curl. A and D are conservative, I accepted both.

8. Solid S is a cylinder $x^2 + y^2 = 4$ bounded below by the xy-plane and above by the plane $x + z = 2$. Its volume is given by which integral?

Ans(8) B or C

A. $\int_0^{2\pi} \int_0^2 \int_0^{2-2\cos\theta} dz\, dr\, d\theta$ **B.** $\int_0^{2\pi} \int_0^2 \int_0^{2-2\sin\theta} dz\, r\, dr\, d\theta$
C. $\int_0^{2\pi} \int_0^2 \int_0^{2-2\cos\theta} dz\, r\, dr\, d\theta$ **D.** $\int_0^{2\pi} \int_0^2 \int_0^{2-x} dz\, r\, dr\, d\theta$

C was the correct answer but by symmetry of the object, B would also be correct so I accepted both.

Part II: Free response. Show all work and be neat! Any work you want looked at for grading purposes must be written within the space provided for the problem.

9. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle 2x + y, y^2 + 3 \rangle$ and C is the line from (1,2) to (4,8).

Not conservative, not a loop.

$\mathbf{r}(t) = \langle 1 + 3t, 2 + 6t \rangle, 0 \le t \le 1.$ $\mathbf{r}'(t) = \langle 3,6 \rangle$

$\mathbf{F}(\mathbf{r}(t)) = \langle 2(1 + 3t) + (2 + 6t), (2 + 6t)^2 + 3 \rangle = \langle 4 + 12t, 7 + 24t + 36t^2 \rangle.$

$\mathbf{F} \cdot d\mathbf{r} = \langle 4 + 12t, 7 + 24t + 36t^2 \rangle \cdot \langle 3,6 \rangle = 3(4 + 12) + 6(7 + 24t + 36t^2) = 216t^2 + 180t + 54.$

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 (216t^2 + 180t + 54)dt = [72t^3 + 90t^2 + 54t]_0^1 = 216.$$

10. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle 4y + e^x, x + \cos(2y) \rangle$ and C is the circle $x^2 + y^2 = 36$ traversed clockwise.

$\mathbf{F}$ is not conservative, C is a loop oriented clockwise, so we'll negate the answer at the end.

$N_x - M_y = 1 - 4 = -3$, Green's Theorem gives $\iint_R (-3)dA = -3 \iint_R dA = -3(\text{area of circle}) = -3(\pi(6)^2)$, so we have -108π. Negated, it is 108π.

11. Solid S is bounded below by the paraboloid $x^2 + y^2 = 1$ and above by the paraboloid $8 - x^2 - y^2$. Set up a triple integral in cylindrical coordinates that gives the volume of S. Do not solve, just set it up.

$$\int_0^{2\pi} \int_0^2 \int_{r^2}^{8-r^2} dz\, r\, dr\, d\theta$$

12. Let $\mathbf{F}(x, y) = \langle 4x^3y^2,\ 2x^4y \rangle$. Suppose path C is a five-sided non-regular polygon.

a) Evaluate $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$, where C_1 is two adjacent sides of the polygon, from (0,0) to (1,0) to (3,2).

F is conservative, potential function is $f(x, y) = x^4y^2$, so evaluate at endpoints, $[x^4y^2]_{(0,0)}^{(3,2)} = 3^4 2^2 = 324$.

b) Using your answer from part (a), what would be the value of $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$, where C_2 is the other three sides of the polygon from (3,2) to two other points and then back to (0,0)?

The line integral around the whole polygon will be 0, so the line integral around the three remaining sides will be –324.