MAT 265 – Exam 2 – Surgent – October 26, 2023 KEY Name:______________________________

Show all work and be neat. You may use the backside for scratch, but work written on the backside won't be looked at. Place all work you want graded on the front. You agree to abide by all aspects of the honor code while taking this exam and will not discuss the contents of this exam during the examination period.

PART I: Multiple Choice. Write the letter of your choice in the space provided at right. (7 pts each)

1. Find the derivative of $f(x) = e^{3x}\ln 2x$.

$$\frac{d}{dx}f(x) = (e^{3x})\left(\frac{1}{2x}2\right) + (3e^{3x})(\ln 2x)$$

Ans(1) B

A. $f'(x) = \frac{e^{3x}}{2x} + 3e^{3x}\ln 2x$ **B.** $f'(x) = \frac{e^{3x}}{x} + 3e^{3x}\ln 2x$

C. $f'(x) = \frac{e^{3x}}{x} + e^{3x}\ln 2x$ **D.** $f'(x) = \frac{3e^{3x}}{2x}$

2. Determine $\lim_{x\to\infty}\frac{4-2x^2}{5+x^2}$.

Ans(2) A

A. -2 **B.** $\frac{4}{5}$ **C.** ∞ **D.** -4

3. Determine $\lim_{x\to 2}\frac{2x^3-7x^2+4x+4}{x^2-4x+4}$.

Do l'Hopital's Rule twice

Ans(3) C

A. ∞ **B.** 0 **C.** 5 **D.** $\frac{0}{0}$

4. Let $y = e^{3x}$, $x = 2$ and $dx = 0.01$. Find dy.

$$dy = 3e^{3x}dx \to dy = 3e^{3(2)}(0.01) = 12.10$$

Ans(4) D

A. 403.43 **B.** 4.03 **C.** 1210.27 **D.** 12.10

5. Find $\frac{dy}{dx}$ where $xy^2 + 3x = 2y^3$.

$$(x)\left(2y\frac{dy}{dx}\right) + (1)(y^2) + 3 = 6y^2\frac{dy}{dx} \quad\to\quad 2xy\frac{dy}{dx} - 6y^2\frac{dy}{dx} = -3 - y^2 \quad\to\quad \frac{dy}{dx}(2xy - 6y^2) = -3 - y^2$$

Ans(5) B

A. $\frac{dy}{dx} = \frac{2xy+y^2+3}{6y^2}$ **B.** $\frac{dy}{dx} = \frac{y^2+3}{6y^2-2xy}$ **C.** $\frac{dy}{dx} = \frac{2xy+3}{6y^2}$ **D.** $\frac{dy}{dx} = \frac{3}{6y^2-2xy}$

6. Find the derivative of $g(x) = x^4 \tan^{-1}(x^3)$

$$\frac{d}{dx}g(x) = (x^4)\left(\frac{1}{1+(x^3)^2}(3x^2)\right) + 4x^3\tan^{-1}(x^3)$$

Ans(6) A

A. $\frac{dg}{dx} = \frac{3x^6}{1+x^6} + 4x^3\tan^{-1}(x^3)$ **B.** $\frac{dg}{dx} = \frac{x^4}{1+x^6} + 4x^3\tan^{-1}(x^3)$

C. $\frac{dg}{dx} = \frac{3x^6}{\sqrt{1-x^6}} + 4x^3\tan^{-1}(x^3)$ **D.** $\frac{dg}{dx} = \frac{x^4}{\sqrt{1-x^6}} + 4x^3\tan^{-1}(x^3)$

7. Give the linearization of $y = 3x^2 + 5x + 2$ at $x = 1$.

$$y' = 6x + 5 \rightarrow \text{at } x = 1, y = 10 \text{ and } y' = 11 \rightarrow y - 10 = 11(x-1) \rightarrow y = 11x - 1$$

Ans(7) D

A. $y = 6x + 5$ **B.** $y' = 11$ **C.** $y = 11x$ **D.** $y = 11x - 1$

8. State the extreme points of $y = 2x^3 - 3x^2 - 36x - 10$.

Ans(8) C

A. $(-2.00001, 34)$ max, $(2.999999, -91)$ min **B.** $(-2, 34)$ min, $(3, -91)$ max **C.** $(-2, 34)$ max, $(3, -91)$ min **D.** $x = -2$ max, $x = 3$ min

Part II: Free response. Show all work and be neat! Any work you want looked at for grading purposes must be written within the space provided for the problem.

9. Use logarithms to find $\frac{dy}{dx}$ where $y = \frac{(x^3+1)^5 e^{6x}}{\sqrt{\sin(2x)}}$. Your answer should be entirely in terms of x.

$$\ln y = \ln(x^3+1)^5 + \ln e^{6x} - \ln\sqrt{\sin 2x}$$

$$\ln y = 5\ln(x^3+1) + 6x - \frac{1}{2}\ln(\sin 2x)$$

Differentiate:

$$\frac{1}{y}\cdot\frac{dy}{dx} = 5\left(\frac{3x^2}{x^3+1}\right) + 6 - \frac{1}{2}\left(-\frac{2\cos 2x}{\sin 2x}\right)$$

$$\frac{dy}{dx} = y\left(\frac{15x^2}{x^3+1} + 6 + \frac{\cos 2x}{\sin 2x}\right) = \frac{(x^3+1)^5 e^{6x}}{\sqrt{\sin(2x)}}\left(\frac{15x^2}{x^3+1} + 6 + \frac{\cos 2x}{\sin 2x}\right)$$

10. Find the linearization of $y = \sqrt{x}$ at $x = 64$ and use it to estimate a value for $\sqrt{66}$.

$$y' = \frac{1}{2\sqrt{x}} \rightarrow \text{at } x = 64, y = 8 \text{ and } y' = \frac{1}{16}$$

Point-slope equation of a line:

$$y - 8 = \frac{1}{16}(x - 64) \quad \rightarrow \quad y - 8 = \frac{1}{16}x - 4 \quad \rightarrow \quad y = \frac{1}{16}x + 4$$

At $x = 66$ we have

$$\sqrt{66} \approx \frac{1}{16}(66) + 4 = 8\frac{1}{8} = 8.125$$

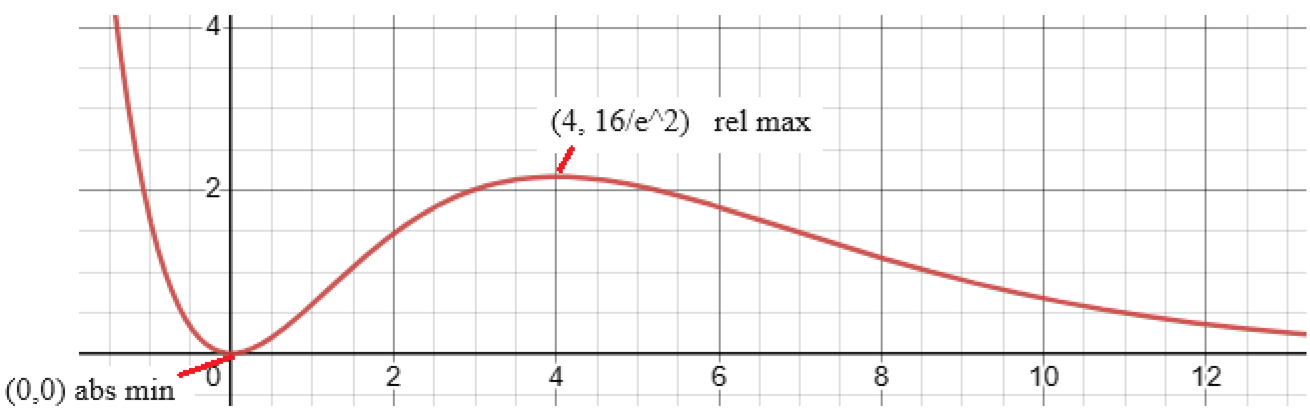
11. Let $f(x) = x^2 e^{-0.5x}$. Use calculus to determine its minimum and maximum points. Give full coordinates in exact form (no decimal equivalents). Give a detailed sketch of the graph with the points properly labeled as relative or absolute minimum or maximum.

$$f'(x) = (x^2)(-0.5e^{-0.5x}) + (2x)(e^{-0.5x}) \quad \rightarrow \quad f'(x) = e^{-0.5x}\left(2x - \frac{1}{2}x^2\right).$$

Set the derivative to 0:

$$e^{-0.5x}\left(2x - \frac{1}{2}x^2\right) = 0 \quad \rightarrow \quad 2x - \frac{1}{2}x^2 = 0 \quad \rightarrow \quad x\left(2 - \frac{1}{2}x\right) = 0 \quad \rightarrow \quad x = 0, x = 4$$

Points: (0,0) abs min & $(4, 16e^{-2})$ rel max.



12. The surface area of a sphere is given by $A(r) = 4\pi r^2$ where r is the radius of the sphere. If the radius of the sphere is changing at a rate of -0.25 cm per minute, what is the rate of change of the surface area of the sphere when the radius of the sphere is 10 cm? Give an exact answer (no decimal equivalents) along with the proper units.

$$\frac{dA}{dt} = 8\pi r\frac{dr}{dt} \quad \rightarrow \quad \frac{dA}{dt} = 8\pi(10)(-0.25) = -20\pi \text{ cm}^2/\text{min}$$