MAT 265 – Exam 1 – Surgent – September 14, 2023 KEY

PART I: Multiple Choice. Write the letter of your choice in the space provided at right. (7 pts each)

1. Find $\lim_{x\to 1}\frac{x^2-1}{2x^3-2x^2}=\frac{0}{0}$. Factor: $\frac{x^2-1}{2x^3-2x^2}=\frac{(x+1)(x-1)}{2x^2(x-1)}=\frac{x+1}{2x^2}$. Thus, $\lim_{x\to 1}\frac{x+1}{2x^2}=\frac{(1)+1}{2(1)^2}=\frac{2}{2}=1$.

   Ans(1) B

   A. -1 **B.** 1 **C.** 0 **D.** Does not exist

2. An object's position, in meters, is given by $s(t)=13+\frac{1}{3}t^3+\sqrt{t}$. What is its instantaneous velocity at $t=4$ seconds? Differentiate: $s'(t)=t^2+\frac{1}{2\sqrt{x}}$. Thus, $s'(4)=(4)^2+\frac{1}{2\sqrt{4}}=16+\frac{1}{4}=16.25$

   Ans(2) C

   A. 18 m/s **B.** 36.33 m/s **C.** 16.25 m/s **D.** 29.25 m/s

3. The limit $\lim_{h\to 0}\frac{(2+h)^3-8}{h}$ represents the derivative of $f(x)$ at a. What are $f(x)$ and a?

   Ans(3) B

   A. $f(x)=x^3, a=8$ **B.** $f(x)=x^3, a=2$ **C.** $f(x)=3x^2, a=8$ **D.** $f(x)=3x^2, a=2$

4. Let $P(t)=\frac{64t+27}{32+4t}$ be the population of bacteria, in thousands, after t minutes. Find the population of this sample as $t\to\infty$. Look at the ratio of the leading coeffs, 64/4 = 16

   Ans(4) D

   A. Infinite **B.** 2000 **C.** 4000 **D.** 16,000

5. Find $\frac{dg}{dt}$, where $g(t)=3\sin(t)+2\cos(t)$

   Ans(5) A

   A. $\frac{dg}{dt}=3\cos(t)-2\sin(t)$ **B.** $\frac{dg}{dt}=3\cos(t)+2\sin(t)$
   C. $\frac{dg}{dt}=-3\cos(t)-2\sin(t)$ **D.** $\frac{dg}{dt}=-3\cos(t)+2\sin(t)$

6. Find $\frac{df}{dx}$, where $f(x)=(x^3+2x)(x^2-1)$. Multiply: $f(x)=x^5+x^3-2x$, then differentiate.

   Ans(6) C

   A. $\frac{df}{dx}=(3x^2+2)(2x)$ **B.** $\frac{df}{dx}=x^5+x^3-2x$ **C.** $\frac{df}{dx}=5x^4+3x^2-2$ **D.** $\frac{df}{dx}=6x^3+4x$

7. Where is the function $f(x)=\frac{5x^2}{(x+2)(x-4)}$ continuous?

   Ans(7) C

   A. $x=-2,4$ **B.** $x=0,-2,4$ **C.** $(-\infty,-2)\cup(-2,4)\cup(4,\infty)$ **D.** $(-\infty,-4)\cup(-4,2)\cup(2,\infty)$

8. Let $f(x)=\begin{cases}x^3-1, & \text{for } x<2\\ ax+3, & \text{for } x\geq 2\end{cases}$. Find a so that f is continuous at $x=2$. Both graphs meet at (2,7)

   Ans(8) A

   A. 2 **B.** 7 **C.** Not possible **D.** 4

Part II: Free response. Show all work and be neat! Any work you want looked at for grading purposes must be written within the space provided for the problem.

9. Let $f(x) = \frac{x^2+4x-21}{x-3}$.

First evaluation: $\lim_{x\to 3} \frac{x^2+4x-21}{x-3} = \frac{0}{0}$, so factor and try again: $\frac{x^2+4x-21}{x-3} = \frac{(x+7)(x-3)}{x-3} = x+7 \quad (x \neq 3)$.

Thus, $\lim_{x\to 3}(x+7) = 10$.

a) Find the following limits.

$\lim_{x\to 3^-} f(x) = 10$ $\qquad$ $\lim_{x\to 3^+} f(x) = 10$ $\qquad$ $\lim_{x\to 3} f(x) = 10$

b) Is f continuous at $x = 3$? Why or why not? No, $f(3)$ does not exist

c) Find $f'(x)$. Leave answer in simplified form. State any restrictions on x.

Since $f(x) = \frac{x^2+4x-21}{x-3} = x+7$, then $f'(x) = 1$ as long as $x \neq 3$.

10. Let $f(x) = x^3 - 3x + 1$.

a) Find the equation of the tangent line, in $y = mx + b$ format, at $x = -2$.

Point: $f(-2) = (-2)^3 - 3(-2) + 1 = -8 + 6 + 1 = -1$. Thus, we have $(-2, -1)$.
Slope: $f'(x) = 3x^2 - 3$, so at $x = -2$, we have slope $f'(-2) = 3(-2)^2 - 3 = 9$.
Using the point-slope equation of the line, we get:

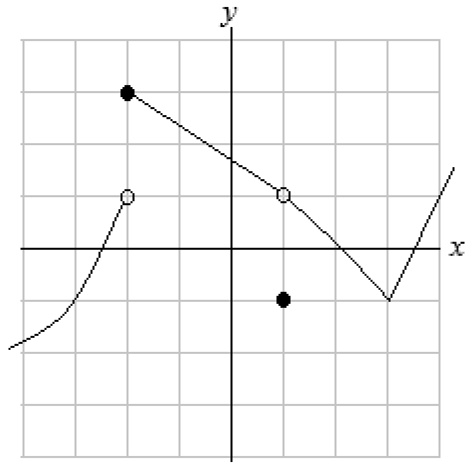
$$y - (-1) = 9(x - (-2))$$
$$y + 1 = 9(x + 2)$$
$$y + 1 = 9x + 18$$
$$y = 9x + 17.$$

b) For what x-values does the function f have tangent lines with slope = 0?

Set the derivative to 0:

$$3x^2 - 3 = 0$$
$$3x^2 = 3$$
$$x^2 = 1$$
$$x = \pm 1.$$

11. Let $y = f(x)$ be the function shown in the graph below. Assume each grid is 1 unit, and the origin is in the exact center. Write DNE if the limit or point does not exist.



a) Find these limits:

$\lim_{x \to -2^-} f(x) = 1$ $\quad$ $\lim_{x \to -2^+} f(x) = 3$ $\quad$ $\lim_{x \to -2} f(x) = DNE$

$\lim_{x \to 1^-} f(x) = 1$ $\quad$ $\lim_{x \to 1^+} f(x) = 1$ $\quad$ $\lim_{x \to 1} f(x) = 1$

b) List the x-values for which the derivative does not exist. $x = -2, 1, 3$

c) Find $f'(0)$. The line passing through $x = 0$ has slope $-\frac{2}{3}$, so $f'(0) = -\frac{2}{3}$

12. Given $\lim_{h \to 0} \frac{5(a+h)^2 - 5a^2}{h}$. Use algebra to simplify and then find the limit. Show all steps.

Simplify:

$$\frac{5(a+h)^2 - 5a^2}{h} = \frac{5(a^2 + 2ah + h^2) - 5a^2}{h}$$
$$= \frac{5a^2 + 10ah + 5h^2 - 5a^2}{h}$$
$$= \frac{h(10a + 5h)}{h}$$
$$= 10a + 5h$$

Take the limit:

Thus, $\lim_{h \to 0} \frac{5(a+h)^2 - 5a^2}{h} = \lim_{h \to 0}(10a + 5h) = 10a$.