MAT267 Quiz 11-9-23 **Name: KEY**

Be neat. Submit to Canvas site by midnight on Nov 10th. One page PDF only.

1. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y)=\langle 6x^2y^5+3,\ 10x^3y^4\rangle$, and C is a line segment starting at (1,1) to (3,4), then another line segment from (3,4) to (5,2), then another line segment from (5,2) to (4,2).

[2.5 pts]

The vector field is conservative, and its potential function is $f(x,y)=2x^3y^5+3x$. Thus, by the Fundamental Theorem of Line Integrals,

$$\int_C \mathbf{F}\cdot d\mathbf{r}=[2x^3y^5+3x]_{(1,1)}^{(4,2)}=\left(2(4)^3(2)^5+3(4)\right)-\left(2(1)^3(1)^5+3(1)\right)=4108-5=4103.$$

2. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y)=\langle 3y,x^2\rangle$, and C is a triangle traced from (0,0) to (3,0) to (3,6) back to (0,0).

[2.5 pts]

The vector field is not conservative, and C is a loop traced counterclockwise (positive orientation), so by Green's Theorem, we have an integrand of $N_x-M_y=2x-3$. The bounds are $0\le y\le 2x$ and $0\le x\le 3$:

$$\text{Green's Theorem: } \int_0^3\int_0^{2x}(2x-3)\,dy\,dx.$$

$$\text{Inner Integral: } \int_0^{2x}(2x-3)\,dy=[(2x-3)y]_0^{2x}=(2x-3)2x=4x^2-6x.$$

$$\text{Outer Integral: } \int_0^3(4x^2-6x)\,dx=\left[\frac{4}{3}x^3-3x^2\right]_0^3=\frac{4}{3}(3)^3-3(3)^2-0=36-27=9.$$

3. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y)=\langle e^{2x}+2y,5x+\sqrt{y^2+1}\rangle$, and C is a rectangle traced from (1,1) to (6,1) to (6,4) to (1,4) back to (1,1).

[2.5 pts]

The vector field is not conservative, so by Green's Theorem, the integrand is $N_x-M_y=5-2=3$. The path C is a loop traced counterclockwise and is a rectangle of length 5 and width 3:

$$\text{Green's Theorem: } \iint_R 3\,dA=3\iint_R dA=3(\text{Area of } R)=3(15)=45.$$

4. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y)=\langle y^2,2xy\rangle$, and C is an ellipse with center (3,8), semi-major axis radius of 6, and semi-minor axis radius of $\sqrt{7}$.

[2.5 pts]

The vector field is conservative, the path is a closed loop, so the line integral is 0.