MAT267 Quiz 11-2-23 **Name: KEY**

Be neat. Submit to Canvas site by midnight on Nov 3rd. One page PDF only.

1. Show that the vector field $\mathbf{F}(x, y) = \langle 3x^2y^4 + 2x,\ 4x^3y^3 - 5y^4 \rangle$ is conservative, then find its potential function.

[3 pts]

If $M = 3x^2y^4 + 2x$, then $M_y = 12x^2y^3$
If $N = 4x^3y^3 - 5y^4$, then $N_x = 12x^2y^3$
Since $M_y = N_x$, the vector field is conservative.

$\int M\, dx = \int 3x^2y^4 + 2x\, dx = x^3y^4 + x^2$.
$\int N\, dy = \int 4x^3y^3 - 5y^4\, dy = x^3y^4 - y^5$.

Thus, the potential function is the union of elements, $f(x, y) = x^3y^4 + x^2 - y^5$.

2. Calculate $\int_C 2xy^2\, ds$ where path C is the straight line from (1,2) to (4,0).

[3 pts]

Parameterize the line: $\mathbf{r}(t) = \langle 1,2 \rangle + t\langle 3,-2 \rangle = \langle 1 + 3t,\ 2 - 2t \rangle$. Thus, $x(t) = 1 + 3t$ and $y(t) = 2 - 2t$. The bounds are $0 \le t \le 1$.

$ds = |\mathbf{r}'(t)| = |\langle 3,-2 \rangle| = \sqrt{3^2 + (-2)^2} = \sqrt{13}\, dt$.

Thus, $\int_C 2xy^2\, ds = \int_0^1 2(1 + 3t)(2 - 2t)^2\sqrt{13}\, dt$.

Foil out the integrand and simplify. The integral is $2\sqrt{13}\int_0^1 (12t^3 - 20t^2 + 4t + 4)\, dt$.

Integrating: $2\sqrt{13}\left[3t^4 - \frac{20}{3}t^3 + 2t^2 + 4t\right]_0^1 = 2\sqrt{13}\left(3 - \frac{20}{3} + 2 + 4\right) = \frac{14}{3}\sqrt{13}$.

3. Calculate $\int_C xy^3\, ds$ where path C is the quarter circle of radius 4, centered at the origin, from (4,0) to (0,4).

[4 pts]

Parameterization: $\mathbf{r}(t) = \langle 4\cos t, 4\sin t \rangle, 0 \le t \le \frac{\pi}{2}$. Thus, $ds = |\mathbf{r}'(t)| = |\langle -4\sin t, 4\cos t \rangle| = 4\, dt$.

Thus, $\int_C xy^3\, ds = \int_0^{\pi/2} (4\cos t)(4\sin t)^3\, 4\, dt = 1024\int_0^{\pi/2} \cos t \sin^3 t\, dt$
$= 1024\left[\frac{1}{4}\sin^4 t\right]_0^{\pi/2} = 1024\left(\frac{1}{4}\right) = 256$.