MAT267 Quiz 10-26-23 **key** **Name:**

Be neat. Submit to Canvas site by midnight on Oct 27th. One page PDF only.

1. Set up a triple integral $\iiint_S f(x,y,z)\,dV$ in the $dz\,dy\,dx$ ordering where S is the solid in the first octant bounded by the *xy* plane, the *xz* plane, the *yz* plane, and the paraboloid $z = 9 - x^2 - y^2$. Just set it up.

[3 pts]

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{9-x^2-y^2} f(x,y,z)\ dx\,dy\,dx.$$

2. Rewrite the integral in problem 1 in cylindrical coordinates and find the volume of the solid.

[3 pts]

$$\int_0^{\pi/2} \int_0^3 \int_0^{9-r^2} 1\,dz\,r\,dr\,d\theta$$

$$= \int_0^{\pi/2} \int_0^3 (9-r^2)r\,dr\,d\theta = \int_0^{\pi/2} \int_0^3 (9r - r^3)\,dr\,d\theta.$$

$$\int_0^3 (9r - r^3)\,dr = \left[\frac{9}{2}r^2 - \frac{1}{4}r^4\right]_0^3 = \frac{81}{2} - \frac{81}{4} = \frac{81}{4}.$$

$$\text{Thus, } \int_0^{\pi/2} \left(\frac{81}{4}\right) d\theta = \frac{81}{4}\left(\frac{\pi}{2}\right) = \frac{81}{8}\pi.$$

3. Evaluate the following integral by first rewriting it in spherical coordinates.

[2 pts rewrite, 2 pts solution]

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (3 + x^2 + y^2 + z^2)\,dz\,dy\,dx.$$

This is 1/8 of a sphere, radius 1, confined to the first octant. Bounds are $0 \le \rho \le 1, 0 \le \theta \le \frac{\pi}{2}, 0 \le \phi \le \frac{\pi}{2}$.

$$\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 (3 + \rho^2)\,\rho^2 \sin\phi\ d\rho\,d\theta\,d\phi.$$

Inner is: $\int_0^1 (3+\rho^2)\,\rho^2 d\rho = \int_0^1 (3\rho^2 + \rho^4)\,d\rho = \left[\rho^3 + \frac{1}{5}\rho^5\right]_0^1 = 1 + \frac{1}{5} = \frac{6}{5}$.

Middle is $\int_0^{\pi/2} \left(\frac{6}{5}\right) d\theta = \frac{3\pi}{5}$.

Outer is $\frac{3\pi}{5}\int_0^{\pi/2} \sin\phi\,d\phi = \frac{3\pi}{5}[-\cos\phi]_0^{\pi/2} = \frac{3\pi}{5}[-0-(-1)] = \frac{3\pi}{5}$.