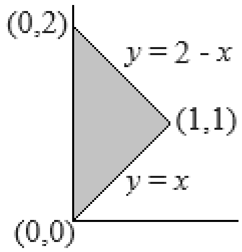
MAT267 Quiz 10-13-23 **KEY** **Name:**

Be neat. Submit to Canvas site by midnight on the date noted above. One page PDF only.

1. Find the volume under the surface $f(x,y) = x^2y$ over the region R in the *xy*-plane enclosed by a triangle with points (0,0), (1,1) and (0,2). [2 pts: set up the double integral, 1 pt: solution]



$$\int_0^1 \int_x^{2-x} x^2y\, dy\, dx$$

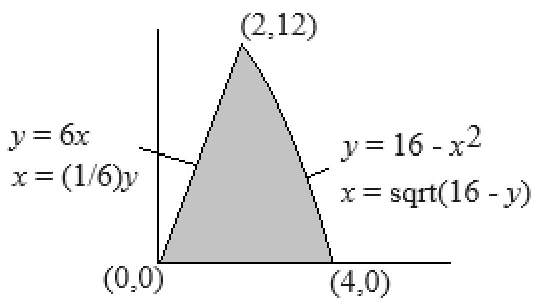
Inner: $\int_x^{2-x} x^2y\, dy = x^2\left[\frac{1}{2}y^2\right]_x^{2-x} = \frac{1}{2}x^2((2-x)^2 - x^2) = 2x^2 - 2x^3.$

Outer: $\int_0^1 (2x^2 - 2x^3)\, dx = \left[\frac{2}{3}x^3 - \frac{1}{2}x^4\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$

2. Rewrite the given compound double integral in the *dx dy* ordering. Re-use $f(x,y)$ as the integrand. Just set it up, do not solve. You are being graded on setting it up correctly. *Hint: sketch the region carefully!*

[3 pts]

$$\int_0^2 \int_0^{6x} f(x,y)\, dy\, dx + \int_2^4 \int_0^{16-x^2} f(x,y)\, dy\, dx.$$

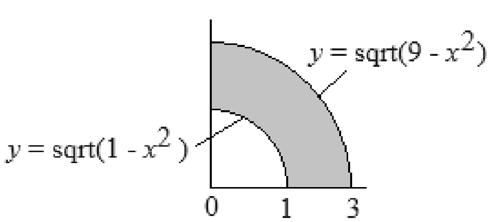


$$\int_0^{12} \int_{y/6}^{\sqrt{16-y}} f(x,y)\, dx\, dy$$

3. Rewrite the given compound double integral using polar coordinates and solve it. *Hint: sketch the region fastidiously and meticulously.*

[4 pts]

$$\int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{9-x^2}} (x^2 + y^2)\, dy\, dx + \int_1^3 \int_0^{\sqrt{9-x^2}} (x^2 + y^2)\, dy\, dx.$$



$$\int_0^{\pi/2} \int_1^3 r^2\, r\, dr\, d\theta = \int_0^{\pi/2} \int_1^3 r^3\, dr\, d\theta$$

Inner: $\int_1^3 r^3\, dr = \left[\frac{1}{4}r^4\right]_1^3 = \frac{1}{4}(3^4 - 1^4) = \frac{1}{4}(81 - 1) = \frac{80}{4} = 20.$

Outer: $\int_0^{\pi/2} 20\, d\theta = 10\pi.$