MAT267 Quiz 10-5-23 **KEY** **Name:**

Be neat. Submit to Canvas site by midnight tonight. One page PDF only.

1. Find all critical points (with full (x, y, z) coordinates) and classify them as minimum, maximum or saddle, of the function $f(x, y) = x^3 + y^3 - 12x - 27y + 2$.

[5 pts]

$f_x = 3x^2 - 12,\ \ f_y = 3y^2 - 27.$ Set to 0: $\begin{matrix} 3x^2 - 12 = 0 \\ 3y^2 - 27 = 0 \end{matrix} \rightarrow \begin{matrix} 3x^2 = 12 \\ 3y^2 = 27 \end{matrix} \rightarrow \begin{matrix} x = \pm 2 \\ y = \pm 3 \end{matrix}$

Critical points are (2,3,-68), (-2,3,-36), (2,-3,40) and (-2,-3,72) (2 pts to here)

D-test: $f_{xx}f_{yy} - f_{xy}^2 = (6x)(6y) - 0.$

At $x = 2, y = 3, D > 0, f_{xx} > 0$ so (2,3,-68) is a minimum

At $x = -2, y = 3, D < 0,$ so (-2,3,-36) is a saddle

At $x = 2, y = -3, D < 0,$ so (2,-3,40) is a saddle

At $x = -2, y = -3, D > 0, f_{xx} < 0$ so (-2,-3,70) is a maximum (3 pts for rest)

2. Use optimization techniques to find the point on the plane $3x + y + 4z = 6$ closest to the point (1,2,3).

[5 pts]

$d = \sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2}$. Let $y = 6 - 3x - 4z$.

Substitute: $d = \sqrt{(x-1)^2 + (6-3x-4z-2)^2 + (z-3)^2} = \sqrt{(x-1)^2 + (4-3x-4z)^2 + (z-3)^2}$

Differentiate:

$$d_x = \frac{2(x-1) + 2(4-3x-4z)(-3)}{2\sqrt{(x-1)^2 + (4-3x-4z)^2 + (x-3)^2}} = \frac{10x + 12z - 13}{\sqrt{(x-1)^2 + (4-3x-4z)^2 + (z-3)^2}}$$

$$d_z = \frac{2(4-3x-4z)(-4) + 2(z-3)}{2\sqrt{(x-1)^2 + (4-3x-4z)^2 + (z-3)^2}} = \frac{12x + 17z - 19}{\sqrt{(x-1)^2 + (4-3x-4z)^2 + (z-3)^2}}$$

Set to 0. Only the numerators are of interest:

$$\begin{matrix} 10x + 12z - 13 = 0 \\ 12x + 17z - 19 = 0 \end{matrix} \rightarrow \begin{matrix} 10x + 12z = 13 \\ 12x + 17z = 19 \end{matrix} \rightarrow \text{solve the system} \rightarrow \begin{matrix} x = -7/26 \\ z = 17/13 \end{matrix}$$

Thus, $y = 6 - 3\left(-\frac{7}{26}\right) - 4\left(\frac{17}{13}\right) = \frac{41}{26}$

I'll accept (-0.269, 1.308, 1.577) but the above work must be shown.

I'll also accept the method of Lagrange multipliers.