**MAT267 Quiz 9-28** **KEY** **Name:**

Be neat. Submit to Canvas site by midnight tonight. One page PDF only.

1. Let $f(x,y) = x^4y^3$ and let $x_0 = 2$ and $y_0 = 3$.

   a) Find the slope of the tangent line at $(x_0, y_0)$ when facing toward the point (4,0).

[2pts]

Direction vector is $v = \langle 2, -3\rangle$ so the unit direction vector is $\frac{1}{\sqrt{13}}\langle 2,-3\rangle = \langle \frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}}\rangle$.

Gradient vector is $\nabla f = \langle 4x^3y^3, 3x^4y^2\rangle$ so $\nabla f(2,3) = \langle 864, 432\rangle$.

Slope is $\nabla f(2,3) \cdot \langle \frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}}\rangle = 864\left(\frac{2}{\sqrt{13}}\right) + 432\left(-\frac{3}{\sqrt{13}}\right) = \frac{432}{\sqrt{13}} \approx 119.82$

   b) Find the direction of the steepest ascent at $(x_0, y_0)$.

[2pts]

$$\nabla f(2,3) = \langle 864, 432\rangle$$

   c) Find the slope of steepest ascent at $(x_0, y_0)$.

[2pts]

$$|\langle 864, 432\rangle| = \sqrt{864^2 + 432^2} = \sqrt{933{,}120} \approx 965.98$$

2. At a quarry, a conical pile of gravel is being created, where 150 m$^3$ of gravel is being added per hour to the pile from the top (so that the pile keeps the conical shape). The volume of a cone of base radius $r$ and height $h$ is $V(r,h) = \frac{1}{3}\pi r^2 h$. Find the approximate change in the height, $\Delta h$, at the moment the height is 10 m and the base radius is 12 m, and where the change in the base radius is $\Delta r = 0.5$ meters/hr.

[4pts]

$$\Delta V = V_r\Delta r + V_h \Delta h$$

$$\Delta V = \frac{2}{3}\pi r h \Delta r + \frac{1}{3}\pi r^2 \Delta h$$

$$150 = \frac{2}{3}\pi(12)(10)(0.5) + \frac{1}{3}\pi(12)^2\Delta h$$

$$150 = 40\pi + 48\pi\Delta h$$

$$\frac{150 - 40\pi}{48\pi} = \Delta h$$

$$0.161 \text{ m/hr} \approx \Delta h$$