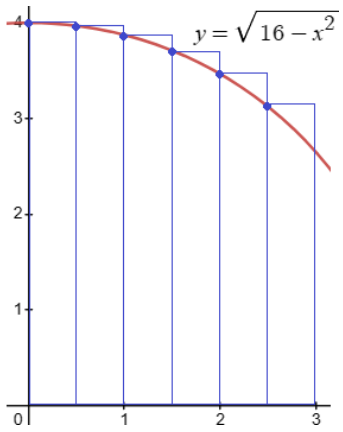
MAT265 Quiz 11/16/23 **Name: key**

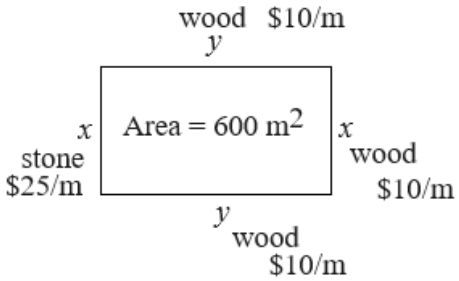
1. Estimate the value of $\int_0^3 \sqrt{16-x^2}\,dx$ using 6 subdivisions of the interval using left rectangles. Give all values to three decimal place accuracy. [3 pts]

Set up a table. Note that each rectangle has width = 0.5 unit.



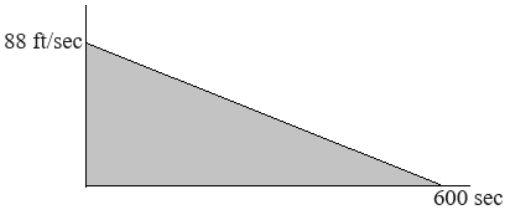
Interval	Left Endpoint	Height = $f(x)$	Area of Rectangle
[0, 0.5]	0	4	0.5 x 4 = 2
[0.5, 1]	0.5	3.969	0.5 x 3.969 = 1.985
[1, 1.5]	1	3.873	0.5 x 3.873 = 1.937
[1.5, 2]	1.5	3.708	0.5 x 3.708 = 1.854
[2, 2.5]	2	3.464	0.5 x 3.464 = 1.732
[2.5, 3]	2.5	3.123	0.5 x 3.123 = 1.562
			Sum = 11.07

2. A rancher plans to enclose a rectangular field covering 600 square meters. One side will be stone fencing costing \$25 per meter to build, and the other three sides will be wooden fencing costing \$10 per meter to build. Find the dimensions that minimize the cost to enclose the field and state the cost too. [3 pts]



Let x be one side, y be the other. So the area is $xy = 600$, or $y = \frac{600}{x}$. Assume one of the x lengths is the stone side. The cost to fence it is $25x$. The cost to fence the other three sides is $10y + 10y + 10x$, so that the total cost to fence the field is $C = 25x + 10y + 10y + 10x$, or $C = 35x + 20y$. To get this into one variable, replace y with $\frac{600}{x}$: we have $C(x) = 35x + 20\left(\frac{600}{x}\right) = 35x + \frac{12000}{x}$. The derivative is $C'(x) = 35 - \frac{12000}{x^2}$. When set to 0, we have $35 - \frac{12000}{x^2} = 0$. Solving for x, we have $x^2 = \frac{12000}{35} \rightarrow x = \sqrt{\frac{12000}{35}} \approx 18.516$ m. This is one side of the rectangle. The other side is $y = \frac{600}{18.516} \approx 32.404$ m, and the cost is $C(18.516) = 35(18.516) + \frac{12000}{18.516} \approx$ \$1296.15.

3. A train moving at 60 miles per hour (88 feet per second) applies its brakes and slows linearly to a full stop after 10 minutes (600 seconds). How far did the train travel during this time? Draw a graph and use geometry to justify your answer. Leave your answer in miles (hint: 1 mile = 5,280 feet). [2 pts]



The graph shows a triangle with base 600 and height 88. Using geometry, the area is $\frac{1}{2}(600)(88) = 26{,}400$ feet. In miles, that is $\frac{26{,}400}{5{,}280} = 5$ miles,

4. An object dropped from a height of 176.4 meters takes exactly 6 seconds to fall under gravity alone, its height above ground given by $f(t) = 176.4 - 4.9t^2$.

   a) Find the average velocity of the object during these 6 seconds. [1 pt]

$$\text{Avg velocity} = \frac{f(6) - f(0)}{6 - 0} = \frac{0 - 176.4}{6} = -29.4\frac{\text{m}}{\text{s}}$$

   b) Find the time t at which the object's instantaneous velocity matched the average velocity found in (a). [1 pt]

Instantaneous velocity is given by $f'(t) = -9.8t$, so set this to -29.4:

$$-9.8t = -29.4 \rightarrow t = \frac{-29.4}{-9.8} = 3 \text{ sec.}$$