MAT265 Quiz 10/19/23 **KEY** **Name:**

Show work, be neat. Submit to the Canvas site by midnight tonight, one page PDF. Original work only. Beware, sites like wolfram, desmos, etc, show things in unique ways and we'll know if you used it.

1. Let $f(x) = x^2 e^{-0.25x^2}$. Use calculus to determine its minimum and maximum points. Show all steps. Give a detailed sketch with these points properly labeled as relative or absolute min or max. Give exact coordinates, no decimal approximations.

[5 pts]

$$y' = x^2\left(-0.5xe^{-0.25x^2}\right) + 2xe^{-0.25x^2}. \text{ Simplified: } y' = e^{-0.25x^2}(-0.5x^3 + 2x)$$

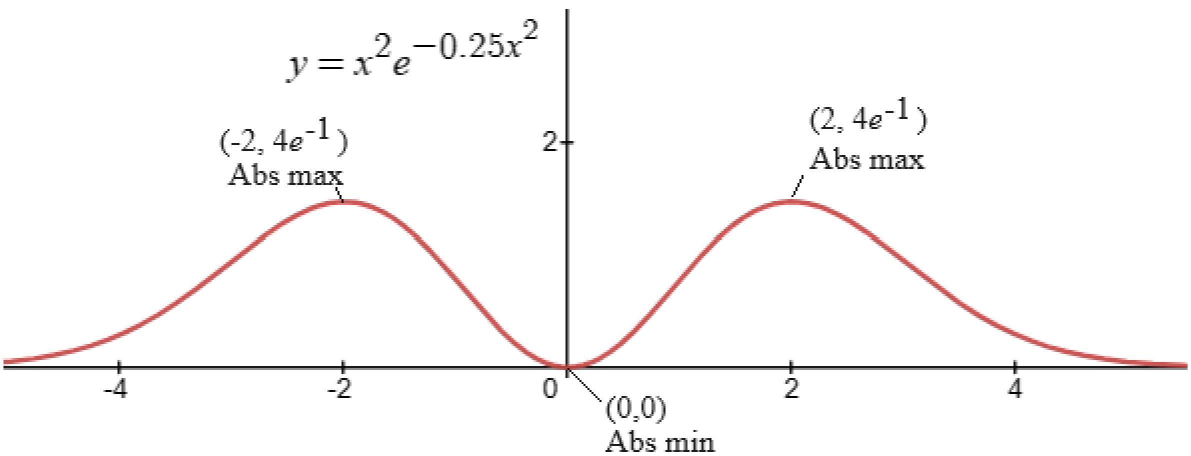
Set to 0. Note that the only solutions will come from $(-0.5x^3 + 2x)$:

$$e^{-0.25x^2}(-0.5x^3 + 2x) = 0 \rightarrow -0.5x^3 + 2x = 0 \rightarrow x(-0.5x^2 + 2) = 0.$$

One solution is $x = 0$. The others are when $-0.5x^2 + 2 = 0$:

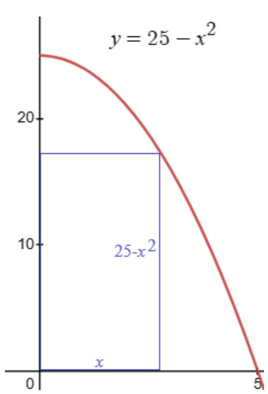
$$-0.5x^2 + 2 = 0 \rightarrow (mult\ by\ 2)\ -x^2 + 4 = 0 \rightarrow x^2 - 4 = 0 \rightarrow x = \pm 2.$$

The critical points are $(-2, 4e^{-1})$, $(0,0)$ and $(2, 4e^{-1})$.



2. A rectangle is to be fitted beneath the curve $y = 25 - x^2$ such that its lower-left corner is at the origin and the upper right corner is on the graph. Use calculus to find the largest possible area of such a rectangle. Give the dimensions of the rectangle and its area.

[5 pts]



The rectangle has base x and height $25 - x^2$, so its area is
$A = (\text{base})(\text{height}) = x(25 - x^2) = 25x - x^3$. Differentiate: $A'(x) = 25 - 3x^2$.
Set to 0 gives $25 - 3x^2 = 0 \rightarrow x^2 = \frac{25}{3} \rightarrow x = \sqrt{\frac{25}{3}} = \frac{5}{\sqrt{3}}$. Thus, the base is $\frac{5}{\sqrt{3}}$, the height is $25 - \left(\frac{5}{\sqrt{3}}\right)^2 = 25 - \frac{25}{3} = \frac{50}{3}$ and the maximum area is
$A = \left(\frac{5}{\sqrt{3}}\right)\left(\frac{50}{3}\right) = \frac{250}{3\sqrt{3}} \approx 48.1125$ sq. units.