MAT265 Quiz 10-17-23 KEY

Differentiate

1. $y = \sin^{-1}(x^2)$

$$y' = \frac{1}{\sqrt{1-(x^2)^2}}(2x) = \frac{2x}{\sqrt{1-x^4}}$$

2. $y = \tan^{-1}(e^{2x})$

$$y' = \frac{1}{1+(e^{2x})^2}(2e^{2x}) = \frac{2e^{2x}}{1+e^{4x}}$$

3. $y = x^2 \sin^{-1}(3x)$

$$y' = x^2\left(\frac{3}{\sqrt{1-(3x)^2}}\right) + 2x\sin^{-1}(3x)$$

4. Use l'Hopital's Rule to find this limit exactly. Show all steps.

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^x$$

Nat log both sides, get into quotient form: $y = \left(1+\frac{3}{x}\right)^x \to \ln y = x\ln\left(1+\frac{3}{x}\right) \to \ln y = \dfrac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{x}}$.

Note that $\lim_{x\to\infty}\left[\dfrac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{x}}\right] = \dfrac{\infty}{\infty}$. Use l'Hopital's: $\lim_{x\to\infty}\left[\dfrac{\frac{1}{1+\frac{3}{x}}\left(-\frac{3}{x^2}\right)}{-\frac{1}{x^2}}\right]$

Simplify: $\dfrac{\frac{1}{1+\frac{3}{x}}\left(-\frac{3}{x^2}\right)}{-\frac{1}{x^2}} = \dfrac{3}{1+\frac{3}{x}}$. Take limit: $\lim_{x\to\infty}\left[\dfrac{3}{1+\frac{3}{x}}\right] = 3$.

Since $\ln y = 3$, base e both sides. Thus, $\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^x = e^3$.