MAT265 Quiz 10/5/23 **KEY** **Name:**

Show work, be neat. Submit to the Canvas site by midnight tonight, one page PDF. Original work only. Beware, sites like wolfram, desmos, etc, show things in unique ways and we'll know if you used it.

1. Let $f(x) = 4e^{x^2+3x}$.

   a) Find the equation of the tangent line of f at $x = 0$.

[3 pts]

$$f'(x) = 4(2x+3)e^{x^2+3x}, \ \ so\ f'(0) = 12 = m = \text{slope.}$$
$$\text{Point is } \big(0, f(0)\big) = (0,4).$$
$$y - y_1 = m(x - x_1) \to y - 4 = 12(x - 0) \to y = 12x + 4.$$

   b) Use the tangent line in part (a) to estimate the value of $f(0.01)$.

[2 pts]

$$f(0.01) \approx 12(0.01) + 4 = .12 + 4 = 4.12.$$

2. Let $g(x) = \frac{e^{4x}(x^4+2x+5)}{\sqrt{\sin x}}$. Use logarithmic differentiation to find $\frac{dg}{dx}$. The final answer must be written in terms of x. Show all steps.

[5 pts]

$$\text{Simplify first:} \quad \ln g(x) = \ln\left(\frac{e^{4x}(x^4+2x+5)}{\sqrt{\sin x}}\right)$$
$$\ln g(x) = \ln e^{4x} + \ln(x^4+2x+5) - \ln\sqrt{\sin x}$$
$$\ln g(x) = 4x + \ln(x^4+2x+5) - \frac{1}{2}\ln(\sin x)$$
$$\text{Differentiate:} \quad \frac{1}{g(x)}g'(x) = 4 + \frac{4x^3+2}{x^4+2x+5} - \frac{1}{2}\left(\frac{\cos x}{\sin x}\right)$$
$$g'(x) = g(x)\left(4 + \frac{4x^3+2}{x^4+2x+5} - \frac{1}{2}\left(\frac{\cos x}{\sin x}\right)\right)$$
$$g'(x) = \frac{e^{4x}(x^4+2x+5)}{\sqrt{\sin x}}\left(4 + \frac{4x^3+2}{x^4+2x+5} - \frac{1}{2}\left(\frac{\cos x}{\sin x}\right)\right)$$